

Practice and Exploration of “Anchored Instruction” of Advanced Mathematics under the Background of Emerging Engineering

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Abstract: Advanced mathematics is an important part of university mathematics, which is characterized by great difficulty and long learning period. If the knowledge concepts learned by students are overly simplified, that is, the knowledge concepts can not reflect the complex situations of real life, the students will find it hard to transfer the knowledge into procedural knowledge for effective problem-solving, and will also struggle to understand the problem-solving methods and thinking patterns of teachers. In response to the demand for talents in the "Emerging Engineering" field, based on the theory of situational cognition, taking the teaching of the second important limit in advanced mathematics as an example, this paper discusses the teaching of advanced mathematics course with anchored instruction model. The teaching practice shows that the anchored instruction model can improve the classroom teaching effect and promote the effective learning for students, which can produce positive teaching effects.

1. Introduction

The courses set for the training objectives of various science and engineering majors must be closely aligned and interconnected with the training objectives. Basic mathematics courses provide the essential knowledge required for many subsequent professional courses and are compulsory subjects for students pursuing postgraduate studies, thus becoming the required courses for most majors. Since February 2017, the Ministry of Education has actively promoted the construction of "Emerging Engineering," requiring higher education institutions to cultivate innovative and outstanding engineering talents who are capable of adapting to the diversified of future society and industries. The introduction of the "Emerging Engineering" talent cultivation program has posed new requirements for the construction of university mathematics courses. Advanced mathematics, which takes functions as research object and limits as its research tool and method, is an important theoretical foundation for learning knowledge in various modern disciplines and a powerful tool for solving practical problems. The syllabus of “Advanced Mathematics A(I)” (June 2023 edition) of

Beijing Institute of Petrochemical Technology (hereinafter referred to as our university) clearly states that Advanced Mathematics A(I) is a compulsory basic course for all science and engineering majors, aimed at cultivating students' mathematical thinking, calculation, expression, and abilities for modeling, and laying the necessary mathematical foundation for subsequent professional courses and professional technical work.

The quality of talent cultivation is the core indicator of higher education quality. The expansion of enrollment over the past 20 years has transitioned our country's higher education from an elite stage to a popularized stage. This change has, on the one hand, provided society with more specialized talents, thereby promoting the rapid socio-economic development and on the other hand, satisfied the intrinsic demand of a larger population for higher education. However, this has also been accompanied by a certain decline in educational quality, leading to continuous reform in university course teaching. The improvement of the teaching model for advanced mathematics has always been one of the directions that university mathematics teachers strive for, and how to carry out effective classroom teaching design is the key. Taking the anchor-based teaching practice carried out by the authors in the teaching of Advanced Mathematics A(I) as an example, based on the theory of situated cognition, combined with the current teaching situation and students' learning situation, this article explores effective methods to help students deeply understand relevant knowledge, stimulate their learning interest, and enhance classroom teaching effectiveness by solving practical problems.

2. Analysis of the Current Teaching Situation

Mathematics courses are an important part of the undergraduate education system, and are crucial for cultivating students' logical thinking abilities and their ability to use mathematics to solve problems. The target audience for Advanced Mathematics A at our university is first-year students in science and engineering. It is their first mathematics course in university, and the class hours run through the entire first-year learning process. It is characterized by a large number of knowledge points and relatively complex relationships between them. Compared with high school mathematics, the difficulty increases, the calculation processes are tedious, and the problems are abstract, which poses a huge challenge to students in their learning.

On the other hand, considering the influence of objective factors such as the teaching syllabus and teaching hours, in actual teaching teachers often do not provide detailed theoretical proofs of some important theorems or calculation formulas in the course, but only explain how to use these theorems or calculation formulas to solve problems. This often leads to students having a superficial understanding of the meaning of the theorems or calculation formulas, and being unable to use them flexibly or using them incorrectly. For students, they have just entered university in the first semester. Although they have already undergone elementary mathematics training in high school, they still lack the psychological preparation for university study, and are basically in a state of passively receiving knowledge, and their learning has a certain degree of blindness. Some students have a fear of difficulty in learning mathematics, and therefore cannot quickly adapt to the university learning method. The highly logical and abstract nature of the course itself makes it easy for students to feel bored and tired during learning, and their learning interest and motivation decrease. Therefore, Advanced Mathematics is also a course with a high failure rate in universities.

3. Situational Cognition Theory

Situational cognition theory is an important learning theory formed in the mid-to-late 1980s. The pioneering work of this theory originated from the "Situated Cognition and Learning Culture" published by Brown, Collins, Duguid, et al. in the journal "Educational Researcher" in 1989, which

argued knowledge is situated, is a part of activities, backgrounds, and cultural products, and knowledge is used and developed in rich contexts^[1]. At the same time, scholars such as J. Lave also proposed the concept of situated learning from an anthropological perspective, proposing the idea that knowledge should be presented in real contexts and requires social interaction and collaboration^[2]. And then the concept of situated cognition was discussed from a psychological perspective, thereby promoting a deeper and more extensive study of situated cognition and learning^[3]. Since then, research on situated cognition has continued to deepen.

There are mainly the following three teaching models guided by situated cognition theory: the anchored instruction model, the random entry teaching model, and the cognitive apprenticeship teaching. The anchored instruction model was proposed by the Cognition and Technology Group at Vanderbilt University led by Professor Bransford. This model focuses on using multimedia technology to present a real and interesting story context to attract learners and bring them into a complex problem context. Identifying the problem in the context is figuratively compared to "anchoring". This model plays an important role in teaching of theoretical knowledge and concepts in university teaching^[4]. Anchored instruction is an educational technology model based on the field of cognitive science. This teaching method makes students generate the need for learning by placing students in a complete authentic problem context. Through embedded instruction and interaction and communication among members of a learning community, namely cooperative learning, students rely on their own active learning and generative learning to personally experience entire process from identifying goals to proposing and achieving them. This teaching method follows two principles: the curriculum design allows learners to explore the teaching content; and both learning and teaching activities revolve an "anchor"—a certain type of case study or problem context^[5].

4. Implementation and Discussion of Anchored Instruction in Advanced Mathematics Teaching

We take the teaching of the second important limit in Advanced Mathematics^[6] as an example.

Using the financial returns of internet financial tools as the "anchor", an anchor-based introduction is used to guide students to analyze problems, explore and solve them. Current internet financial tools have brought unprecedented convenience to people's lives. More and more people use internet finance for payment or wealth management, and this type of financial management allows investors to see their returns every day. Internet finance is a compound interest model. So, the more rolling interest is calculated within the same storage period, will the final sum of principal and interest be greater? To what extent will the final sum of principal and interest be greater? This further leads to the problem of compound interest in real life.

Assume that the annual interest rate of a certain internet financial instrument is 100%, and someone invests 10,000 yuan, calculates the sum of principal and interest one year later according to compound interest. The teachers may organize students to perform calculations with annual, semi-annual, quarterly and other interest calculation periods as interest calculation periods, and finally obtain the table 1^[7].

Table 1: Calculation of the sum of principal and interest for different interest periods.

Interest calculation period	Interest rate within one interest calculation period	Sum of principal and interest after one year (Unit: 10,000 yuan)
Annual	1	$1+1=2$
Semi-annual	$1/2$	$(1+1/2)^2 = 2.25$
Quarterly	$1/4$	$(1+1/4)^4 \approx 2.44$

Monthly	1/12	$(1+1/12)^{12} \approx 2.613$
Weekly	1/52	$(1+1/52)^{52} \approx 2.6926$
Daily	1/365	$(1+1/365)^{365} \approx 2.7146$
hourly	1/8760	$(1+1/8760)^{8760} \approx 2.7181$
minutely	1/525600	$(1+1/525600)^{525600} \approx 2.71828$

From the table, it can be seen that the shorter the interest calculation period, the greater the sum of principal and interest returned at the end of the year. If interest is calculated n times a year, the interest each time is $1/n$. If 10,000 yuan is invested, the sum of principal and interest at the end of the year is $(1+1/n)^n$ ten thousand yuan. This triggers students' thinking and forms a positive cognitive atmosphere. Question 1: Does the sum of principal and interest after one year increase infinitely as the interest calculation period decreases? Question 2: After one year, will the investor become a "millionaire"? By converting these two questions into the problems of sequence limits, the situation is gradually internalized into knowledge through activities. Question 1: Is the sequence

$\left\{ \left(1 + \frac{1}{n} \right)^n \right\}$ monotonically increasing? Question 2: If it is monotonically increasing, will there be

$\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n} \right)^n = \infty$? The teacher explained that it can be proven that $(1+1/n)^n$ increases with the increase of n , but will never exceed 3. According to the Limit Existence Criterion II, when n approaches positive infinity, the limit of $(1+1/n)^n$ exists, which is denoted as

$$\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n} \right)^n = e.$$

Furthermore, combined with the research history of e , students can establish an emotional connection with mathematics and deepen their understanding and memory. When the famous mathematician Jacob Bernoulli was studying continuous compound interest, he realized that it needed to be described by the limit value of the $\{(1+1/n)^n\}$. Combined with the compound interest problem, the meaning of e is given: the limit value that can be reached by the continuous doubling growth of unit of principal within a unit of time. And in 2004, Google went public with an IPO, and the fundraising amount of the initial public offering was 2718,281,828 US dollars, which is the first ten digits of e .

Further compare the sum of principal and interest after one year under the cases of infinite interest calculation and annual interest calculation. Assume that the annual interest rate of a certain internet financial instrument is x , 10,000 yuan is invested, and interest is compound. If interest is calculated once a year, the sum of principal and interest after one year is $1+x$. If interest can be calculated infinitely many times a year, the sum of principal and interest after one year is

$$\lim_{n \rightarrow +\infty} \left(1 + \frac{x}{n} \right)^n = e^x.$$

For example, when $x=5\%$, the sum of principal and interest after one year with continuous compounding and annual compounding result in a total principal and interest of approximately 1.05127 ten thousand yuan and 1.05 ten thousand yuan, respectively. The maximum difference between the two is 12.7 yuan, the gap is not as large as imagined. If the principal is k , the total principal and interest after t years with continuous compounding is ke^{at} . Thus, extending from e the exponential function $y=ke^{at}$, based on the meaning of e , it can be used to calculate the quantity of anything that continuously increases or decreases at a constant rate a time t , such as capital

accumulation, bacterial growth, etc.

The theoretical proof of the second important limit is relatively complex, making it difficult for students to understand and accept in the classroom. The instructional design of this lesson avoids the theoretical proof and adopts an anchored instruction model, treating knowledge as an application tool. This provides a more authentic and meaningful way of learning, and promotes the transformation of knowledge into real-life contexts. Furthermore, through authentic contexts and a series of activities related to the knowledge, students are helped to internalize the knowledge. Students participate in authentic activities, transfer their knowledge, and ultimately achieve the active construction of knowledge.

5. Conclusions

Advanced mathematics has an important and far-reaching impact on cultivating students' rational thinking, forming their scientific literacy, and improving their ability to analyze and solve problems. The exploration of teaching models by university mathematics teachers is of great significance. Preliminary practice shows that the anchored instruction model can not only complete students' knowledge structure and deepen their desire for knowledge, but also effectively promote teacher-student interaction, increase students' participation, stimulate students' learning, and further enhance the teaching effect in the classroom.

Educational reform is the driving force for the continuous development and progress of education. The reform of university mathematics courses is a long-term and systematic subject, as well as a very arduous task. How to continuously optimize teaching models and promote students' effective learning still requires mathematics educators to continuously explore and seek, in order to provide strong support for students to achieve their learning goals and realize comprehensive development.

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