DOI: 10.23977/aetp.2026.100112
ISSN 2371-9400 Vol. 10 Num. 1

High School Mathematics Teaching Design Exploration Based on the Core Competency of Visual-Spatial Thinking-Taking the Circumscribed Sphere of a Regular Triangular Pyramid as an Example

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***Keywords*:** Visual-Spatial Thinking; Core Competency; High School Mathematics; Teaching Design; Regular Triangular Pyramid

***Abstract*:** Focused on core competency-oriented education reform, this study explores integrating visual-spatial thinking cultivation into high school mathematics teaching via the "circumscribed sphere of a regular triangular pyramid" case. Grounded in constructivism, it designs targeted strategies (GeoGebra demonstration, hands-on drawing) to address students' spatial learning difficulties, proving concrete teaching vehicles effectively turn abstract competency goals into actionable learning outcomes.

1. Teaching Objectives

1.1 Teaching Applications

This case study has the following teaching applications:

Demonstrating Teaching Practice: It provides front-line high school mathematics teachers, especially new teachers and Master of Education (M.Ed.) students, with a complete and operable demonstration of how to integrate the core competency of "visual-spatial thinking" into specific classroom teaching. It showcases a systematic teaching design centered on a challenging geometry topic: the circumscribed sphere of a regular triangular pyramid.

Providing Teaching Strategies: The case study integrates diverse teaching strategies, including GeoGebra dynamic demonstrations, student hands-on drawing, analogical transfer from plane to space, and tiered practice. It offers specific methodological references for addressing common solid geometry teaching challenges, such as "students' difficulty in spatial imagination, technical drawing, and forming solution frameworks." [1]

Clarifying Design Concepts: This case emphasizes that teaching design should not merely focus on imparting problem-solving routines, but should delve into the geometric essence (e.g., determining the sphere's center) and integrate mathematical thinking methods like the combination of numbers and shapes, and transformation. This helps teachers enhance the depth and conceptual orientation of their teaching design.

Promoting Professional Development: By studying, analyzing, and reflecting on this case, teachers can deepen their understanding of core mathematical competencies (especially visual-spatial thinking) and improve their abilities in curriculum content interpretation, student needs analysis, objective setting, and teaching process design, thereby fostering their own professional growth.

1.2 Target Audience

This case is primarily developed for M.Ed. students, particularly those majoring in Subject Teaching (Mathematics), and can also be used in related courses within the M.Ed. program for Subject Teaching (Mathematics).

1.3 Applicable Courses

Research on Middle School Mathematics Curriculum and Textbooks
Research on Middle School Mathematics Teaching Design and Implementation
Research on Mathematics Education Reform

2. Guiding Questions for Reflection

To promote in-depth understanding and transferable application of this case, the following reflection questions are proposed:

How does this case achieve the transfer of knowledge and methods from the "circumscribed circle of a triangle" to the "circumscribed sphere of a triangular pyramid"? In what other areas of solid geometry teaching could this "plane-to-space" design approach be applied? (e.g., analogizing properties from parallelograms to parallelepipeds)

The case emphasizes using GeoGebra dynamic demonstrations to support teaching. Please consider: When using multimedia technology to facilitate the cultivation of "visual-spatial thinking," how should teachers balance "technological demonstration" with "students' independent imagination and drawing" to prevent technology from replacing students' thinking processes?

The case points out the limitations of simply teaching circumscribed sphere "models." Teachers need to discuss, based on their experience, how to balance "enabling students to master general models to improve problem-solving efficiency" and "guiding students to explore geometric essence to develop thinking abilities" in teaching.

The teaching objectives of this case explicitly target the "visual-spatial thinking" competency. Teachers can try selecting another solid geometry topic (such as "section problems of prisms" or "tangency and inscription problems between spheres and polyhedra") and, based on the design ideas of this case, briefly outline the key points of a teaching design centered on cultivating the core competency of visual-spatial thinking.

3. Analysis Framework

3.1 Starting Point: Top-Level Core Competency Goals

Core Competency: Visual-Spatial Thinking. This serves as the starting point and ultimate goal of the entire instructional design. All teaching activities are designed to foster this core competency.

3.2 Focus: Core Teaching Problems

The case accurately identifies three core difficulties students face when learning solid geometry (especially circumscribed sphere problems), forming the "problem orientation" of the instructional design:

Problem 1 (Spatial Imagination Difficulty): Students cannot construct, rotate, and analyze complex 3D figures in their minds.

Problem 2 (3D Drawing Difficulty): Students cannot accurately translate their spatial conceptions onto paper, hindering problem analysis and solving.

Problem 3 (Solution Strategy Formulation Difficulty): When faced with comprehensive problems, students do not know where to start and struggle to identify the "key" to the solution.

3.3 Bridge: Instructional Design Theories and Ideas

To address the above problems, the case design is grounded in solid teaching theories and mathematical thinking:

Theoretical Basis: Constructivist Learning Theory. It emphasizes that learning is not passive reception but active construction by the learner. Elements such as "student hands-on drawing," "teacher-student collaborative exploration," and "GeoGebra dynamic perception" are designed to enable students to actively construct the mental schema of the "circumscribed sphere of a regular triangular pyramid."

Thinking Methods: Combination of Numbers and Shapes, and Transformation Ideas

Combination of Numbers and Shapes: Connects abstract parameters (base side length (a), height (h) with intuitive figures (sphere center location, circumscribed sphere), using "shape" to interpret "numbers" and "numbers" to define "shape."

Transformation/Reduction: Transforms the complex spatial problem (finding the sphere's radius) into a familiar plane problem (applying the Pythagorean theorem in a right triangle within the axial cross-section).

3.4 Carrier: Specific Knowledge Point

Core Knowledge Point: The circumscribed sphere of a regular triangular pyramid. This topic is highly comprehensive, a challenging point in college entrance exams, and an excellent vehicle for cultivating spatial imagination. It naturally encompasses positional relationships (the sphere center lies on the height), metric relationships (the relationship between radius (R), (a), and (h), and graphic transformations.

3.5 Key: Teaching Strategies and Practice

Theory, problems, and knowledge are ultimately translated into three interconnected teaching strategies, each corresponding directly to one of the three major problems:

Strategy 1: Dynamic Demonstration, Making the Implicit Explicit (Addresses "Spatial Imagination Difficulty")

Practice: Use GeoGebra to demonstrate the dynamic transition from a triangle's circumscribed circle to a pyramid's circumscribed sphere, showing how the sphere center changes with different values of (a) and (h).

Logical Relationship: Technology visualizes and animates abstract, hidden spatial relationships, directly assisting students in preliminary spatial construction and overcoming "imagination difficulty."

Strategy 2: Hands-on Drawing, Making the Abstract Concrete (Addresses "3D Drawing Difficulty")

Practice: Require students to draw figures independently first, then compare their work with standard diagrams, experiencing a cycle of "imagine-draw-revise-reimagine."

Logical Relationship: Converts the vague "virtual" image in the mind into a tangible "real" object on paper. This is a crucial step in externalizing internal thought, serving as a bridge between "visualization" and "imagination," and overcoming "drawing difficulty."

Strategy 3: Key Element Extraction, Simplifying Complexity (Addresses "Solution Strategy Formulation Difficulty")

Practice: Guide students through observation and exploration to discover and understand the two key geometric quantities determining the circumscribed sphere of a regular triangular pyramid: base side length (a) and height (h).

Logical Relationship: Unifies the complex circumscribed sphere problem around these two fundamental quantities, using the axial cross-section to transform the spatial problem into a planar right-triangle problem (Pythagorean theorem: $R^2 = (h - R)^2 + r^2$; where r is the circumradius of the base). This provides students with a clear and universal problem-solving approach, overcoming "strategy formulation difficulty." [2]

3.6 Endpoint: Achievement of Teaching Objectives

Through this logically rigorous and progressively designed process, the case naturally achieves its pre-set teaching objectives: Students not only master the knowledge and methods related to the circumscribed sphere of a regular triangular pyramid but also enhance their abilities in spatial imagination, technical drawing, and problem analysis – effectively cultivating the core competency of "visual-spatial thinking."

3.7 Summary

This case's analysis framework illustrates the complete logic that an excellent teaching design should possess: guided by core competencies, rooted in genuine student needs, grounded in educational theory and mathematical thinking, using specific knowledge as a vehicle, and ultimately achieving the coordinated development of student abilities and competencies through a series of precise and effective teaching strategies[3]. This analytical framework is not only applicable to this case but also provides a powerful tool for analyzing and designing other teaching cases.

4. Case Analysis

This case represents a well-crafted, theoretically grounded, and highly exemplary teaching design.

Solid Theoretical Core: Its design aligns with modern educational concepts such as cognitive load theory, constructivism, and meaningful learning.

Scientific Teaching Strategies: Through "plane-to-space" transfer, technology application following the principle of "imagine first, verify later," and knowledge construction progressing "from principles to models," it effectively cultivates students' visual-spatial thinking competency.

High Transferability: Its analytical approach (e.g., identifying core geometric quantities, transforming problems into axial section analysis) can be widely applied in teaching other polyhedron-sphere problems, such as circumscribed and inscribed spheres for pyramids, prisms, and other geometric solids.

This case not only teaches us "how to teach" but, more importantly, reveals how a high-quality mathematics lesson should be thoughtfully conceptualized and scientifically designed in the era of core competencies.

5. Summary of Key Points

This case provides profound insights for mathematics teaching oriented towards core competencies, especially in solid geometry:

Implementing Competencies Requires Concrete "Vehicles": Cultivating the core competency of "visual-spatial thinking" cannot be achieved through empty talk; instead, it must rely on challenging and typical specific teaching content such as the "circumscribed sphere of a regular triangular pyramid," and be delivered through carefully designed activities.

Technology Should Serve as a "Scaffold for Thinking": Multimedia technology (like GeoGebra) is not for show or to replace thinking; its optimal role is as a "verifier," "magnifying glass," and "feedback source." It should be introduced after students have engaged in necessary mental struggle, supporting rather than replacing their independent construction process[4].

Model Teaching Should Delve into Geometric Essence: Teaching should explore the geometric essence of models (like the "axial section" in this case), revealing underlying mathematical principles (like the Pythagorean theorem). This allows students to grasp the logic behind model construction rather than merely memorizing model conclusions.

Breaking Through Difficulties Lies in "Effective Transformation": The challenge of solid geometry lies in its "spatial" nature. The success of this case lies in identifying the key: "spatial problem planarization." By utilizing the axial cross-section, the 3D sphere problem is transformed into a 2D right-triangle problem, enabling a cognitive breakthrough.

Teaching Design Should Begin with "Student Analysis" and End with "Competencies": Excellent teaching design must accurately diagnose students' real difficulties (imagination, drawing, strategy formulation) and, guided by these insights, integrate various teaching resources and strategies, ultimately aiming to enhance students' comprehensive competencies. This case's introduction starting from the "plane circumscribed circle" is precisely based on such accurate student analysis.

In conclusion, this case demonstrates how to elevate a traditional "problem-solving skill lesson" into a "thinking training lesson" and a "competency cultivation lesson."[5] It reminds us that the true essence of mathematics teaching is not to teach students how to solve many problems, but to guide them to understand how to think about a class of problems, and in this process, grow into thinkers with richer imagination and logical reasoning ability.

Acknowledgements

Construction of a Teaching Case Database for "Secondary School Mathematics Curriculum and Textbook Research" Based on the New Curriculum Reform. (SDYAL2025)

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