

# *Research on Instructional Design of Junior High School Quadratic Functions Based on PBL Model*

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**Abstract:** This study constructs a PBL-based instructional design for junior high school quadratic functions, targeting three dilemmas in traditional teaching: abstract concept comprehension, low learning motivation, and rigid knowledge application. Through real problem scenarios, analogical concept construction, and diversified evaluation, it realizes student-centered inquiry learning. The design not only helps students master quadratic function concepts and develop mathematical thinking but also provides a replicable framework for integrating PBL into mathematics teaching, highlighting its value in fostering students' core literacy.

## 1. Teaching Objectives

### 1.1 Teaching Purposes

This case has the following teaching applications:

#### 1.1.1 Provide a practical operation example of the PBL model in mathematics

This case demonstrates for master's students in education how to transform abstract PBL theory into an executable teaching process. Through specific problems such as "orchard planting" and "bank savings", it presents the complete operation process from problem proposal to achievement evaluation, helping master's students in education understand the specific implementation methods of PBL in teaching practice.

#### 1.1.2 Demonstrate new ideas for mathematical concept teaching

In traditional teaching, the concept of quadratic functions is often introduced through direct definitions. This case shows how to guide students to experience the complete process of "specific problem → abstract model → concept formation" through problem scenarios, providing a new paradigm for concept teaching for master's students in education<sup>[1]</sup>.

#### 1.1.3 Display effective strategies for classroom organization and management

The case describes in detail links such as group construction, task allocation, and process guidance, providing specific references for master's students in education on how to organize

inquiry-based classrooms and how to play a guiding role in different links.

#### **1.1.4 Improve instructional design ability**

By analyzing this case, master's students in education can learn how to design driving problem scenarios based on curriculum standards, textbook content, and students' actual situation, how to set progressive inquiry tasks, and how to integrate core literacy goals into specific teaching links.

#### **1.1.5 Deepen the understanding of the essence of mathematics teaching**

This case reflects that mathematics teaching should shift from "knowledge imparting" to "thinking cultivation", helping master's students in education understand that the essence of mathematics teaching is to guide students to experience the process of mathematization and cultivate mathematical thinking and problem-solving ability.

In summary, this case has extensive and in-depth teaching applications. It can not only directly serve classroom teaching practice but also promote the professional development of master's students in education. It is a high-quality teaching case with multiple values. It not only tells master's students in education "what to teach", but more importantly, shows "how to teach" and "why to teach in this way", realizing the organic combination of teaching theory and practical operation.

### **1.2 Target Learners**

This case is mainly developed for master's students in education. It is suitable for postgraduate students majoring in subject teaching (mathematics), and can also be used in related courses for master's students in education majoring in subject teaching (mathematics).

### **1.3 Applicable Courses**

Research on Middle School Mathematics Curriculum and Textbooks, Research on Instructional Design and Implementation of Middle School Mathematics, Research on Mathematics Education Reform

## **2. Inspirational Thinking Questions**

To promote in-depth understanding and transfer application of this case, the following thinking questions can be put forward:

### **2.1 The case uses two problems of "orchard planting" and "bank savings" as introductions**

Do you think these two problems constitute high-quality "driving questions"? Please analyze their advantages and limitations combined with the characteristics of driving questions in PBL (such as authenticity, openness, challenge, etc.).

If you are asked to redesign a more attractive driving question around the core concept of "quadratic functions" for local students, how will you design it? Please explain your design ideas and the teaching considerations behind them.

Design Intent:

Guide master's students in education to critically evaluate the core elements of teaching cases. Driving questions are the soul of PBL. This question requires them not only to understand the concept but also to use standards for professional judgment.

Promote knowledge transfer and innovative design, moving from case analysis to independent design, and cultivating their ability to develop curriculum resources based on students' situation and local cultural background.

## 2.2 Under the "student-centered" concept of PBL, the role of teachers is transformed into a guide

In the "group cooperation, exploring concepts" link of this case, at which specific step do you think students may encounter cognitive difficulties? (For example, summarizing common points from specific examples, or making conjectures by analogy with existing knowledge)

In response to the predicted difficulty, as a teacher, what kind of "learning scaffolding" (such as providing guiding question sheets, designing analogy examples, organizing phased reports, etc.) will you build to promote inquiry rather than directly telling the answer? Please describe your intervention strategy.

Design Intent:

Concretize and contextualize the abstract principle of "teacher-oriented". Teacher educators should prompt master's students in education to deeply think about the real learning situation that may occur in PBL practice and plan teaching countermeasures in advance.

Cultivate the ability of precise teaching intervention. Let them understand that "guidance" is not laissez-faire, but requires careful instructional design and keen classroom insight, which is the key to the success of PBL.

The PBL model has obvious advantages in achieving the knowledge goal of "mastering the definition of quadratic functions"<sup>[2]</sup>. However, for subsequent skill-based and understanding-based goals such as "drawing complex quadratic function graphs" or "understanding the influence of coefficients a, b, and c on the graph", do you think PBL is still the most efficient teaching model? Why?

Generally speaking, what key supporting conditions (such as students' foundation, class hour arrangement, school culture, teachers' ability, etc.) are required to successfully implement the teaching described in this case in an ordinary middle school? What may be the biggest challenge?

Design Intent:

Guide master's students in education to view teaching models dialectically, realizing that any model has its applicable boundaries and limitations. Teacher educators should avoid falling into the misunderstanding of "PBL-only", and cultivate their rational thinking of flexibly choosing and integrating teaching methods according to teaching content.

Stimulate practical considerations of the "context" of teaching implementation. Teacher educators should pull the discussion from idealized instructional design back to complex educational reality, think about the impact of systematic factors such as management, resources, and evaluation on teaching reform, and improve their macro thinking and practical planning ability.

## 2.3 Analysis Ideas

### 2.3.1 Starting Point: Accurately Diagnosed Core Teaching Problems (Top-level Core Problems)

The case starts from three common dilemmas in the teaching of junior high school "quadratic functions":

Problem 1 (Abstract concept, difficult to understand): As a leap in thinking from linear to nonlinear, the abstractness of quadratic functions makes it difficult for students to establish accurate mathematical concepts.

Problem 2 (Lack of motivation, low interest): The traditional teaching mode of "definition - graph - property - application" easily makes students feel bored and lack learning motivation.

Problem 3 (Rigid knowledge, difficult to transfer): Although students can recite definitions and formulas, they cannot identify and establish quadratic function models in real scenarios.

These three problems together constitute the "difficulties" that the case needs to solve.

### 2.3.2 Bridge: PBL Model as the Theoretical Core of the Solution

The innovation of the case lies in that it introduces the PBL (Problem-Based Learning) theory as the "brain" and "framework" of top-level design, instead of using technology scattered or improving teaching methods. The three core concepts of the PBL model exactly address the above three core problems:

Concept 1 (Problem-centered): Through real scenarios such as "orchard planting" and "bank savings", abstract mathematical concepts are embedded in specific problems, making knowledge "useful" and "necessary to learn", directly overcoming Problem 1 (abstract concept) and Problem 2 (lack of motivation).

Concept 2 (Student-centered): Students are no longer passive receivers of knowledge, but problem solvers and active constructors of knowledge. Through group cooperation and independent inquiry, students experience the process of concept generation in person, thus achieving in-depth understanding of knowledge and cultivating cooperation and inquiry ability.

Concept 3 (Teacher-guided): Teachers transform from lecturers to designers and guides of learning. By building cognitive scaffolds such as "question chains" and "analogical conjectures", teachers guide the direction of students' inquiry, ensure learning efficiency, avoid students getting lost in independent inquiry, and effectively support the smooth implementation of the PBL model<sup>[3]</sup>.

### 2.3.3 Carrier: Development of Specific Teaching Strategies

Under the guidance of PBL theory, the case transforms concepts into three interlocking teaching strategies, putting theory into practice as executable classroom activities:

Strategy 1: Create real problem scenarios (corresponding to "problem-centered")

Practice: Use "orchard yield increase" and "compound interest calculation" as initial problems.

Logical relationship: These problems act as "cognitive conflicts" to create learning needs. To solve the problems, students must find new mathematical tools, thus naturally arousing the desire to explore the concept of quadratic functions.

Strategy 2: Guide analogical concept construction (corresponding to the combination of "student-centered" and "teacher-guided")

Practice: Guide students to analogize the research paths of linear functions and inverse proportional functions, analyze the common points of examples, and independently summarize the definition and general form of quadratic functions.

Logical relationship: This is the embodiment of PCK (Pedagogical Content Knowledge). Teachers have a deep understanding of the commonalities of function learning (concept - graph - property - application) and design activities for students to re-experience the process of concept formation. This is not only about learning knowledge but also about mastering the general method of studying functions, realizing "teaching someone to fish".

Strategy 3: Hierarchical application and diversified evaluation (corresponding to knowledge transfer and literacy achievement)

Practice: From direct application (paint cost) to complex modeling (sales profit), and adopt a combination of self-evaluation, peer evaluation, and teacher evaluation.

Logical relationship: Through variant exercises, promote the transfer of knowledge in different

scenarios, solving Problem 3 (rigid application). Diversified evaluation shifts the focus from "answers" to "process" and "ability", comprehensively evaluating students' growth in the PBL process, which is consistent with the goal of core literacy<sup>[4]</sup>.

### 2.3.4 End Point: Achievement of Teaching Objectives and Implementation of Literacy

Through the above logically rigorous design, the case finally effectively achieves its multi-dimensional teaching objectives:

Knowledge and skill objectives: Students independently construct and deeply understand the concept of quadratic functions in solving problems.

Process and method objectives: Through the complete inquiry cycle, students' model thinking, reasoning ability, cooperation, and problem-solving ability are systematically trained.

Emotional attitude and values objectives: In the process of solving real problems, students realize the value of mathematics, enhance their interest in learning and application awareness.

In summary, the analysis ideas of this case reveal the internal logic of an excellent PBL instructional design: it starts from a profound insight into real teaching problems, succeeds with a solid educational theory as the overall framework, and finally lands on a series of teaching strategies that seamlessly connect students, problems, and knowledge. This is not only a lesson plan but also a thinking model of "how to scientifically carry out instructional design".

## 3. Case Analysis

This case is a PBL teaching example with advanced concepts, complete design, and great inspiration.

It clearly shows the core process of PBL: starting from real problems, constructing concepts through cooperative inquiry, and applying them to new scenarios.

It reveals the key details of successful implementation: the quality of driving questions and teachers' precise scaffolding support are the "details" that determine the success or failure of PBL.

It triggers rational thinking about teaching models: reminding us to scientifically select and use teaching models according to teaching objectives and the nature of learning content to achieve complementary advantages.

The value of this case lies not only in itself but also in providing a template for master's students in education to criticize, analyze, learn from, and surpass, effectively promoting the development of their professional ability in instructional design.

## 4. Summary of Key Points

This case provides profound enlightenment beyond itself for "mathematics teaching oriented to core literacy":

The essence of PBL lies in "reconstructing the meaning of learning": PBL is not just "learning with problems", but reconstructs students' intrinsic motivation for learning by endowing knowledge with real and urgent application scenarios. It answers the fundamental question in students' minds "Why should I learn this?".

Teachers' core ability shifts from "lecturing" to "designing" and "guiding": In PBL classrooms, teachers' professional ability is reflected in designing high-quality driving questions and providing precise "scaffolds" in the inquiry process. This requires teachers to have a deeper understanding of the essence of the subject and students' cognitive rules.

Knowledge structure is more important than the quantity of knowledge: This case guides students to analogize the research paths of learned functions, infiltrating the universal methodology

of "how to study a function". This structured and transferable thinking method is far more valuable than memorizing the definition of a quadratic function in isolation.

Evaluation is the "navigation system" of learning: The diversified evaluation system in the case guides students' learning attention to literacy dimensions such as cooperation ability, inquiry spirit, and thinking depth. This enlightens us that "what to evaluate" determines "what students will learn to be", and the reform of teaching evaluation is the key to promoting the transformation of teaching methods<sup>[5]</sup>.

Teaching models are "prescriptions" rather than "laws": This case shows the great advantages of PBL in introductory concept courses. However, it also reminds us that PBL is not suitable for all types of courses (such as skill training courses). Excellent teachers should, like doctors, flexibly choose or even mix different teaching models according to different teaching objectives and content.

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Construction of a Teaching Case Database for "Secondary School Mathematics Curriculum and Textbook Research" Based on the New Curriculum Reform. (SDYAL2025)

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