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GARCH-Based VaR Estimation for the INE Crude Oil Futures Market

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Abstract: This paper estimates the Value at Risk (VaR) of Shanghai INE crude oil futures using GARCH models based on 1403 daily data points. The study finds that the INE crude oil futures return series (RETURN) is stationery and exhibits left-skew, leptokurtosis (peaked), and heavy tails, along with weak autocorrelation. Volatility demonstrates a "leverage effect," where bad news generates greater volatility than equivalent good news. At a 95% confidence level, the VaR is 4.634% of the asset's market value, indicating significant risk. Furthermore, the EGARCH-T model is shown to accurately estimate this risk. This research provides a valuable reference for investors in assessing risk and formulating strategies.

1. Introduction

The deepening of global economic integration has heightened the systemic importance of crude oil futures markets as a core segment of international financial markets. As the “lifeblood” of modern industry, crude oil is highly sensitive to a wide range of factors, including geopolitical tensions, macroeconomic and monetary policy shifts, and technological change. These forces jointly drive pronounced volatility and uncertainty in crude oil futures prices, exposing investors, financial institutions, and energy-related firms to substantial market risk. In this context, how to accurately measure and manage the risk of crude oil futures has become a central issue in contemporary finance and energy economics.

Value at Risk (VaR) has emerged as a widely used tool for modern risk management because it provides a single, intuitive metric of the maximum potential loss of a position or portfolio over a given horizon at a specified confidence level. By quantifying potential losses, VaR helps market participants understand and control their risk exposure. However, the crude oil futures market exhibits complex, nonlinear dynamics and pronounced volatility clustering, features that traditional risk measurement methods often fail to capture adequately. As a result, conventional VaR approaches may underestimate or misrepresent the true risk in crude oil futures markets, underscoring the need for more accurate and flexible modelling techniques.

The generalized autoregressive conditional heteroskedasticity (GARCH) framework offers a powerful approach for modelling financial time series with time-varying and clustered volatility. GARCH models explicitly incorporate the dependence of current volatility on past shocks and past volatility, making them particularly well suited to describing the dynamics of crude oil futures prices.

In addition, by allowing for alternative error distributions and model specifications, GARCH-type models can be tailored to the fat tails and asymmetries that frequently characterize energy price returns, thereby improving the precision of risk estimates. Combining GARCH models with VaR estimation integrates modern risk measurement with state-of-the-art volatility modelling, enabling more accurate assessment of crude oil futures market risk at different confidence levels.

Against this backdrop, an empirical study of crude oil futures market risk based on GARCH–VaR has important theoretical implications. First, it deepens our understanding of the risk characteristics and volatility dynamics of crude oil futures by explicitly modelling time-varying risk and revealing the mechanisms through which shocks propagate in energy markets. Second, it enriches the risk management literature by extending the application of modern financial risk tools to a key energy derivative market, improving the accuracy and applicability of risk measurement in this domain. Third, by empirically evaluating the performance of GARCH-based VaR models, the study provides evidence on their effectiveness, offering a solid foundation for further theoretical development in financial risk management.

The research also carries substantial practical significance. More accurate VaR estimates for crude oil futures can help investors and firms quantify their risk exposure and design more rational trading, hedging, and asset allocation strategies, thereby reducing losses arising from blind or poorly informed decisions. For financial institutions, GARCH-based VaR provides a more robust basis for internal risk control, capital allocation, and stress testing. For regulators and policymakers, improved risk metrics can enhance market surveillance, support macroprudential oversight, and contribute to the stable and efficient functioning of energy and financial markets. Therefore, an empirical investigation of crude oil futures market risk using a GARCH–VaR framework is not only of clear academic interest but also of considerable practical relevance and policy value.

2. Theoretical Background

2.1 Research on Value at Risk (VaR)

Value at Risk (VaR) research started earlier in developed capital markets and is now a fundamental component of contemporary risk management. VaR-based frameworks are recommended by international organizations like the Basel Committee and the Bank for International Settlements, and its application in international financial institutions has been standardized thanks to J.P. Morgan's RiskMetrics project. The variance-covariance technique, historical simulation, and Monte Carlo simulation have all been used in later research to improve VaR estimation, increasing its applicability in risk management and portfolio management. At the same time, the fundamental analytical tools for connecting asset returns, risk, and the best possible portfolio selection are provided by contemporary portfolio theory, which is exemplified by Markowitz's mean-variance model, absolute deviation models, the Sharpe single index model, and the capital asset pricing model[1]. VaR research began later but has advanced rapidly in China. In addition to using VaR and VAR models to examine macrofinancial links like inflation transmission and regional economic growth, researchers increasingly utilize VaR to quantify market risk in stock indices and other financial assets.[2]. Although there is certainly need for development, these applications demonstrate that VaR-based techniques may be modified for the Chinese environment[3].

International research on Value at Risk (VaR) started early and has been widely used in risk management. Institutions such as the Bank for International Settlements and the Basel Committee recommend VaR as a core risk measure, and initiatives by J.P. Morgan helped standardize its use in practice[4][5]. Subsequent studies refined historical simulation, Monte Carlo and variance covariance methods, which strengthened the performance of VaR in portfolio management and risk control. At the same time, Markowitz’s mean variance model, absolute deviation models, the Sharpe single index model and the capital asset pricing model provide the main theoretical tools for linking returns and risk and for constructing optimal portfolios[6]. In China, VaR research began later but developed quickly. Scholars apply VaR to measure market risk in Shanghai and Shenzhen stock indices and adapt its calculation under GED distributions and fat tailed returns[7]. Other studies combine VaR with vector autoregression models to analyze price transmission between CPI and PPI and the drivers of regional economic growth, showing that VaR and VAR frameworks are useful for describing the joint behavior of financial risk and macroeconomic variables in the Chinese context[8].

2.2 Related Studies on GARCH Models

GARCH model research started early in international markets and has advanced quickly. A family of extensions, including EGARCH, TGARCH, and GJR GARCH, were developed from early research that concentrated on the fundamental idea of conditional heteroskedasticity. These extensions are currently frequently utilized in risk management, portfolio allocation, and option pricing. According to recent empirical research, arbitrage and asset allocation techniques can outperform market benchmarks in terms of Sharpe ratios and performance by integrating time-varying volatility from GARCH models[9-11]. Building on this body of work, academics in China have modified the GARCH and AR GARCH specifications to fit the features of their own financial markets[12-14]. When applied to statistical arbitrage and pairs trading, these studies typically discover that the basic GARCH (1,1) model performs well in the A share market and that volatility bands derived from GARCH models produce more consistent returns and lower risk than fixed parameter rules.[15-16]

2.3 Research on Crude Oil Price Risk Measurement

Existing studies on crude oil price risk can be broadly divided into qualitative and quantitative approaches. Qualitative work highlights the quasi-financial nature and non-renewable scarcity of oil, arguing that price fluctuations are driven not only by supply and demand but also by geopolitical events, macroeconomic conditions and investor behaviour, and thus examines pricing mechanisms and futures market participation[17-19]. Quantitative studies mainly rely on the GARCH family, VaR frameworks and extreme value theory to model stylised facts of oil returns such as excess kurtosis, fat tails, volatility clustering and leverage effects[20-22]. This literature finds that GED GARCH, long-memory GARCH, EGARCH–EVT–Copula and parametric or semiparametric extreme-value models generally outperform traditional RiskMetrics and simple GARCH specifications in predicting VaR for WTI and Brent markets, especially at high confidence levels, and capture asymmetric risks, regime shifts and the higher risk associated with price increases[23-24]. Building on these contributions, Chinese studies adapt these models to domestic markets such as Daqing, refine threshold selection, volatility bands and pairs-trading rules, and provide additional evidence on crude oil price risk in the Chinese context[25-26].

3. Theoretical Foundation for GARCH Model-Based VaR Calculation

3.1 Introduction to GARCH Models

3.1.1 ARCH Model

Bollerslev introduced the ARCH model in 1982 as an econometric tool to account for heteroscedasticity in time series data. For time series variables, it tackles problems resulting from the second premise of conventional econometrics, which is constant variance. The following is an expression for the ARCH model equation:

$$\varepsilon_t = \sigma_t z_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2$$

In the equation: ε_t denotes the random error term; σ_t^2 denotes the conditional heteroskedasticity term; z_t denotes an independent and identically distributed random variable with mean 0 and variance 1.

3.1.2 GARCH Models

By further quantifying the variance of the error factor, the GARCH model improves on the ARCH model and fits the initial time series data. This method produces more thorough analytical outcomes.

The ARCH(p) model is introduced. Considering that excessively large p-values may result in loss of degrees of freedom, it is extended to GARCH(p,q), which can be expressed as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_q \sigma_{t-q}^2$$

The ARCH model is expanded upon by the GARCH model. Fitting an ARCH model is necessary prior to application in order to determine whether volatility clustering is present in the data. The GARCH model captures heteroskedasticity more thoroughly than the ARCH model because it includes the qth-order lag effects of variance in its analysis. Given its universality, the GARCH(1,1) model is commonly used in financial time series analysis, especially when working with return data.

3.1.3 TGARCH and EGARCH Models

The TGARCH model can be written as follows when negative information generates more volatility than positive information:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i} + \sum_{j=i}^{q} \beta_j \sigma_{t-j}^2 + \sum_{k=i}^{r} \gamma_k \varepsilon_{t-k} d_{t-k}$$

In the equation: $\gamma_k \varepsilon_{t-k} d_{t-k}$ denotes the asymmetric effect term; d_{t-k} denotes an explanatory variable.

Nelson then put forth the EGARCH model, which is based on the GARCH model and has the following equation:

$$\ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left(\left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - E\left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| \right) + \sum_{j=1}^{q} \beta_j \ln(\sigma_{t-j}^2) + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}}$$

3.2 The Method of Value at Risk (VaR)

3.2.1 Definition of Value at Risk

Value at Risk, or VaR, is a crucial metric for assessing the risk of portfolio management. Usually represented numerically as $P(\Delta P > VaR) = 1 - k$, it is the worst-case scenario under given parameters. The VaR approach makes the assumption that assets' future returns will be distributed normally. The desired end-period value, $W = W_0(1 + R)$, is obtained by setting the initial value to W_0 and the expected return rate to R. In practice, however, negative return rates are common. The predicted return rate in these situations needs to be specified at a specific level of confidence. The related minimum intended end-period value is $W^* = W_0 (1 + R^*)$, which is the minimum expected return, represented by the symbol R^*.

The mean of VaR can be written as follows, assuming that VaR_R is the benchmark loss in relation to E(W):

$$VaR_R=E(W)-W^*=-W0(R^*-\mu)$$

The initial value of VaR, if it is specified as the initial value, can be written as follows:

$$VaR_A= W0-W^*=-W0R^*$$

It is evident from the analysis above that the real VaR value is somewhat near to the minimal value W^* and minimum return R^*. It can be solved to the least return R^* within a specified confidence range, which is:

$$\alpha = \int_{R}^{+\infty} f(r)dr$$

In portfolio value analysis, researchers often apply several Value at Risk (VaR) frameworks, including parametric and nonparametric techniques, to analyze model performance under various scenarios. Because it can be applied ex ante across a variety of asset classes, provides quantitative support for risk measurement and centralized risk management in financial institutions, and condenses possible future losses and their probabilities into a single statistic, VaR has emerged as a key tool for assessing portfolio market risk. However, VaR is primarily a quantile-based measure, delivering little information on losses beyond the specified cutoff and failing to satisfy critical coherence features such as subadditivity and convexity, which may lead to risk underestimation in extreme cases. To overcome these limitations, further studies provide conditional value at risk (CVaR) and similar methods that focus greater emphasis on tail losses and provide a more thorough assessment of downside risk.

3.2.2 VaR Model Measurement Methods

The parametric and non-parametric approaches are the two main groups into which the VaR calculation techniques fall. The variance-covariance method, which necessitates knowledge of the return distribution for financial assets or securities portfolios, is the most widely used parametric approach. The two most commonly used non-parametric techniques are Monte Carlo simulation and historical simulation, neither of which requires assumptions regarding the return distribution for stocks portfolios or financial assets.

(1) The Method of Variance-Covariance

The widely used parametric variance-covariance method has long been used to calculate financial risk Value at Risk (VaR). This method models asset portfolio or financial time series returns using the functional relationship between market forces and time series, assuming a normal distribution. The VaR value of a financial time series can be calculated at a certain confidence level using statistical measurements like the distribution's mean, variance, and covariance, enabling parameter estimation and empirical investigation. The variance-covariance approach is popular with financial management firms and economic scholars because to its simplicity, minimal computational requirements, and easy of comprehension. This strategy has drawbacks. such Monte Carlo and historical simulation, it assumes a normal distribution and ignores financial time series characteristics such “heavy tails” and volatility clustering. This assumption contradicts financial data features, which could distort historical financial trend projections. The predicted VaR values may be underestimated or exaggerated, compromising empirical analysis accuracy.

(2) The Historical Simulation Approach

Historical simulation techniques use historical data and previous sample data to forecast future value changes in financial asset portfolios. This method's benefit is that it doesn't rely on the assumption that financial data sequences are normally distributed. It successfully manages nonlinear problems, reduces model risk, and more properly depicts the "heavy-tailed" and asymmetric features of financial time series. Its drawbacks, meanwhile, are that historical data can only represent previous shifts in the market. The accuracy of empirical research may be impacted if future market dynamics diverge from past patterns, which could result in imprecise forecasts of future market trends based on historical data. Notwithstanding these drawbacks, the Basel Accords have accepted the historical simulation method as one of the core techniques for evaluating market risk because of its stability and ease of use.

(3) The Method of Monte Carlo Simulation

Although Monte Carlo simulation is comparable to historical simulation in some ways, it is based on the assumption that financial time series are linear and have a normal distribution. In order to enable further analytical research, models are built using historical data and parameter estimate is carried out to determine VaR values at particular confidence levels. The method's strength is its capacity to do comprehensive scenario simulations for a wide range of market situations and factors, successfully managing nonlinear problems and the "heavy-tailed" phenomena in financial time series. As a result, study findings are more exact, and VaR values for empirical analysis are precise. As a result, Monte Carlo simulation is more accurate and efficient than historical simulation when it comes to determining financial risk VaR values.

Monte Carlo simulation does, however, have some drawbacks. This approach necessitates a large amount of computing in order to produce a broad set of scenarios, which adds model risk while also increasing computational complexity and time consumption. Moreover, Monte Carlo simulations are predicated on the assumptions of a normal distribution and linear features in financial sequences, and they rely on historical data samples, just like historical simulation techniques. This could cause financial sequences to deviate from their actual course, which would impair the precision of later parameter estimation and empirical studies.

3.2.3 Theoretical Structure for Calculating GARCH-VaR

The normal distribution probability density function of the GARCH model, which is used in practice to determine VaR values, can be written as follows:

$$f(x)=\frac{1}{\sqrt{2\pi\sigma}}\exp(-\frac{1}{2\sigma^2}(x-\mu)^2),x\in R$$

In the formula: μ denotes the mean; σ^2 denotes the variance.

The following is an expression for the t-distribution's probability density function:

$$f(x)=\frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})\sqrt{v\pi}}(1+\frac{x^2}{v})^{(\frac{v+1}{2})},x\in R$$

In the formula: Γ（∗） denotes the gamma function; v denotes the degrees of freedom.

The probability density function employing a generalized error distribution can be expressed as:

$$f(x)=\frac{v\exp(-\frac{1}{2}\left|\frac{x}{\lambda}\right|)}{\Gamma(\frac{1}{v})\lambda 2^{(1+\frac{1}{v})}},x\in R$$

In the formula: Γ（∗） denotes the gamma function; v represents the degrees of freedom; λ denotes the tail thickness parameter.

By incorporating the conditional variance into the GARCH model, VaR can be expressed as:

$$VaR = P_{t\text{-}1} Z_{\alpha} \sigma$$

4. Empirical Findings

4.1 Sample Selection

To assess crude oil market trends, this study uses the closing prices of Shanghai INE crude oil futures, which comprise 1,403 daily data points and span the sample period from March 26, 2018, to December 31, 2023. Using information from the Wind database, a GARCH model is created.

4.2 Statistical Feature Analysis

4.2.1 Descriptive Statistics

The trend in the original series is removed and transformed into a return format by log-differencing the closing price. $R_t = lnx_t - lnx_{t-1}$ is the formula used for the computation, and x_t is the closing price of INE crude oil futures on day t. Below are the time series charts for the differentiated return sequence RETURN and the INE crude oil price sequence PRICE, as shown in Figure1.

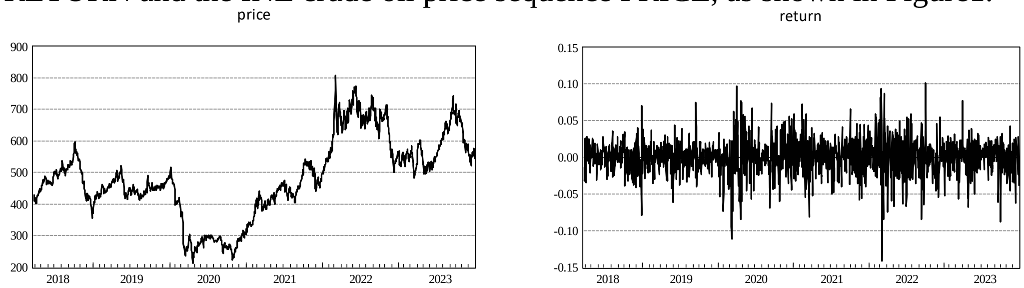

Figure 1 Time Series Chart of INE Original Price Sequence PRICE and Differenced Return Sequence RETURN

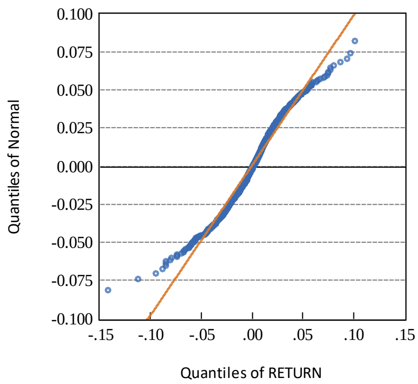

Figure 2 QQ Plot of the Yield Sequence

A QQ (Quantile-Quantile) plot was created in order to examine the return series' distribution properties in more detail, as shown in Figure2. Quantiles of the normal distribution are shown on the vertical axis of this graphic, while quantiles of the return series are shown on the horizontal axis. The scatter plot shows the quantiles of the return distribution, whereas the straight line in the plot depicts the trajectory of a perfect normal distribution. The fit between the actual distribution and the normal distribution is higher when the scatter plot's distribution trend is closer to the straight line. The properties of the distribution, on the other hand, differ from the normal distribution if the scatter deviates from the line. It can be concluded that the return sequence of crude oil futures does not follow a normal distribution since it is evident from observation that the return distribution of these contracts deviates significantly from the normal distribution's reference line. This conclusion is consistent with earlier findings from analysis using the Jarque-Bera (JB) statistic.

In conclusion, the INE crude oil futures returns employed in this work show a leptokurtic, heavy-tailed feature rather than a normal distribution. To better reflect the leptokurtic, heavy-tailed nature of the data, the residuals of the return series should subsequently resemble a T-distribution or GED distribution when the GARCH model is established.

4.2.2 Normality Test

The figure below displays the results of descriptive statistics and normalcy testing.

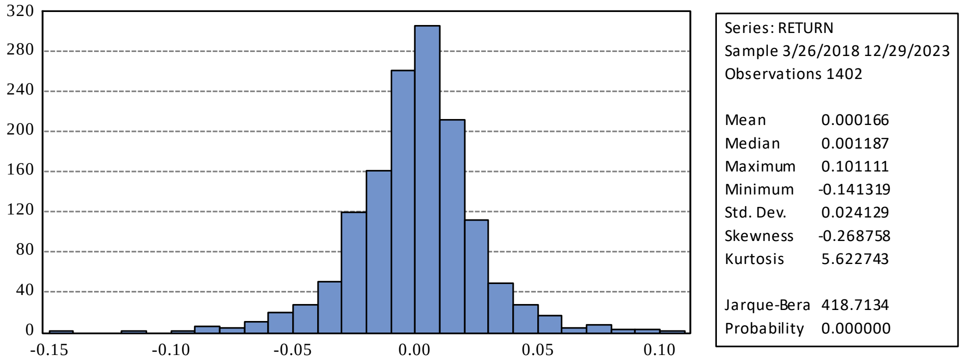

Figure 3 Normality Test

It is evident that the normalcy test mean is nearly zero. The associated skewness value would be 0 if the distribution were normal. However, the preceding table shows a left-skewed distribution with a skewness of -0.268758, which is less than 0. The RETURN sequence exhibits a peaked, heavy-tailed pattern, as shown in Figure 3, with a kurtosis of 5.622743, significantly higher than the kurtosis of a normal distribution (3). The p-value is significantly less than 0.01 and the J-B statistic value is 418.7134. As a result, the null hypothesis that the RETURN sequence follows a normal distribution is rejected, proving that it does not. Rather, it displays a leptokurtic, left-skewed distribution with hefty tails.

4.2.3 Stability Test

With a p-value of 0 ($p < 0.01$) and an ADF unit root test statistic of -37.23880, which is less than the critical value at the 1% significance level, the null hypothesis of a unit root is rejected at the 1% significance level, proving that the RETURN series is a stationary time series, as shown in Table1.

Table 1 Stability Test

Variables		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-37.23880	0.0000
Test critical values:	1% level	-3.434808	
	5% level	-2.863396	
	10% level	-2.567807	

4.2.4 Autocorrelation Test

Autocorrelation testing is necessary because crude oil futures returns show clustering features and may exhibit autocorrelation. The autocorrelation function (ACF) and partial autocorrelation function (PACF) are two tools for measuring autocorrelation. The association between a time series's current value and its past data at particular points in time is reflected by the autocorrelation function. On the other hand, after adjusting for the impact of various lag periods, the partial autocorrelation function shows the relationship between a time series's present value and its past data at particular points in time. The Q statistic, which has N degrees of freedom and a chi-squared distribution, is used for autocorrelation testing. According to the null hypothesis, serial autocorrelation does not exist. In order to better capture the properties of the data, the mean-variance model should include lagged time series terms, as rejecting the null hypothesis suggests that autocorrelation exists in the time series.

Figure 4 displays the autocorrelation test plot for the yield rate that was acquired using the Eviews program.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.004	0.004	0.0180	0.893
		2	0.043	0.043	2.6358	0.268
		3	-0.007	-0.007	2.7053	0.439
		4	0.029	0.027	3.8581	0.426
		5	-0.042	-0.042	6.3647	0.272
		6	-0.035	-0.037	8.0913	0.231
		7	-0.040	-0.035	10.295	0.172
		8	0.017	0.019	10.704	0.219
		9	0.021	0.026	11.331	0.254
		10	-0.071	-0.073	18.479	0.047
		11	0.031	0.029	19.820	0.048
		12	0.032	0.033	21.252	0.047
		13	-0.003	-0.010	21.266	0.068
		14	0.003	0.006	21.276	0.095
		15	0.006	0.003	21.332	0.127
		16	0.004	0.000	21.351	0.165
		17	-0.036	-0.038	23.157	0.144
		18	0.012	0.019	23.378	0.176
		19	-0.037	-0.030	25.316	0.150
		20	0.011	0.002	25.492	0.183
		21	-0.027	-0.019	26.554	0.186
		22	0.037	0.038	28.518	0.159
		23	0.010	0.009	28.662	0.192
		24	-0.012	-0.022	28.873	0.225

Figure 4 Plot of Yields for the Autocorrelation Test

Only the Q-statistics at lags 10, 11, and 12 reject the null hypothesis of no serial autocorrelation at the 5% significance level, as can be shown when lags range from 1 to 24. The null hypothesis is accepted at every other latency level. This suggests that serial autocorrelation exists in the INE crude oil futures return series, albeit weakly.

4.2.5 Heteroscedasticity Test

It is crucial to determine whether the return series is heteroskedastic before constructing the GARCH model. This study builds a mean equation with only one constant term, eliminates the residual series, and applies an ARCH-LM test to it in light of the crude oil return series' weak autocorrelation. The null hypothesis is that the residual series does not exhibit an ARCH effect. It is evident that the chi-squared test's F-statistic and P-value are both far lower than the significance level of 0.01. Therefore, the null hypothesis is rejected, indicating that the residuals exhibit an ARCH effect. As a result, this time series can meet the requirement for GARCH modeling by establishing a conditional variance equation using a GARCH-type model to match the residual sequence's heteroskedasticity, as shown in Table 2.

Table 2 Test for Heteroscedasticity

Heteroskedasticity Test: ARCH			
F-statistic	31.24683	Prob.F(1,1399)	0.0000
Obs*R-squared	30.60787	Prob.Chi-Square(1)	0.0000

4.3 Estimation of GARCH Models

The aforementioned statistical research shows that the returns on crude oil futures show heavy-tailed distributions, volatility clustering, and asymmetry. A model of the ARMA-GARCH type can be developed to forecast VaR and fit returns. This paper uses the most popular GARCH(1,1) model for modeling since higher-order GARCH(p,q) models are prone to multicollinearity and over-parameterization problems. We further expand the GARCH(1,1), EGARCH(1,1), and TGARCH(1,1) models, assuming normal N, T, and GED distributions for the marginal distributions of the return series, respectively, to guarantee fitting quality. This results in nine models that can be used to suit the return series, as shown in Table3.

Table 3 Estimation Results of the GARCH Model

Model	distribution	*c*	*mu*	*α*	*β*	*γ*	*s*	*AIC*	*LLH*
GARCH	*N*	0.000758	0.000021***	0.116628***	0.851112***			-4.746933	3331.599863
	T	0.001246**	0.000016***	0.097743***	0.879023***		6.071014***	-4.784880	3359.200952
	GED	0.001321**	0.000019***	0.104225***	0.866045***		1.362470***	-4.785078	3359.339524
EGARCH	*N*	0.000395	-0.580989***	0.245332***	0.947914***	-0.041908***		-4.748791	3333.902811
	T	0.000999**	-0.476031***	0.210710***	0.958156***	-0.044026**	6.152412***	-4.786571	3361.386457
	GED	0.001124**	-0.531524***	0.224922***	0.952578***	-0.039152**	1.367288***	-4.785957	3360.956139
TGARCH	*N*	0.000485	0.000023***	0.088934***	0.845514***	0.057051***		-4.748981	3334.035779
	T	0.001070**	0.000019***	0.072738***	0.870254***	0.052870*	6.183550***	-4.785796	3360.843174
	GED	0.001158**	0.000021***	0.079746***	0.858974***	0.050504*	1.369102***	-4.785776	3360.829178

Note: ***, **, and * denote significance at the 1%, 5%, and 10% significance levels, respectively.

The ARCH and GARCH coefficients are both significantly positive under the N-, T-, and GED-distribution models, according to the GARCH model's estimation findings. Moreover, the parameter requirements are satisfied since the total and difference of these coefficients are both less than and around 1. The conditional variance of crude oil futures returns shows persistent shocks, as indicated by the convergence of the GARCH model. In other words, shocks have a big impact on all future projections. According to the EGARCH and TGARCH models' estimation results, the GARCH and ARCH coefficients are both substantially positive. The asymmetry coefficient in the TGARCH model is significantly positive, but it is significantly negative in the EGARCH model. This suggests that there is a "leverage effect" at play in the volatility of crude oil futures prices: bearish news might cause more significant swings than an equivalent quantity of bullish news. The EGARCH model under the T-distribution is the best model in terms of model estimate confidence criteria since it produces the lowest AIC value and the highest LLH value. In order to anticipate Value at Risk (VaR) under a given limit, the crude oil futures return series is then fitted using the EGARCH-T model.

By removing residuals from the estimated equations for ARCH testing, the EGARCH-T model's validity was examined. The findings demonstrated that there was no heteroskedasticity in the residuals because the F-statistic and the chi-square statistic P-values were both higher than 0.1. This demonstrates how well the model fits the data, as shown in Table4.

Table 4 ARCH Tests

Heteroskedasticity Test:ARCH			
F-statistic	0.402807	Prob.F(1,1399)	0.5257
Obs*R-squared	0.403266	Prob.Chi-Square(1)	0.5254

4.4 Value at Risk Calculation

The calculation formula is:

$$VaR_t = \mu_t + Z_\alpha \sigma_t$$

The GARCH model's forecast value at time t is represented by μ_t in the equation, whereas Z_α indicates the quantile that corresponds to a given confidence level. With a confidence level of 0.95, the T-distribution in this study yields Z_α= 1.934593. The expected volatility at time t is denoted by σ_t. The period from March 26, 2018, to December 31, 2023, was forecasted using in-sample forecasting and the established model. After obtaining the conditional variance, the square root of volatility was determined. Eviews was used to calculate the VaR value, and descriptive statistics were applied to the results. These are the outcomes, as shown in Table5.

Table 5 VaR Value Descriptive Statistics

Mean	Median	Maximum	Minimum	Std.Dev.	Skewness	Kurtosis
0.04634	0.04435	0.10774	0.02339	0.01260	1.29462	5.99665

The VaR average is 0.04634, indicating that at a 95% confidence level, the potential loss for INE crude oil futures is 4.634% of the asset's market value, representing significant risk.

Plot the crude oil futures yield and VaR trend charts as follows, as shown in Figure5.

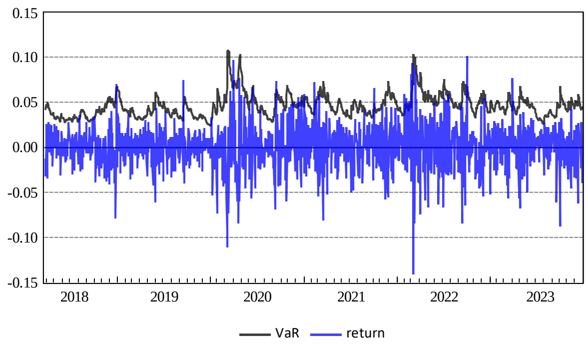

Figure 5 Crude Oil Futures Return and VaR Trend Chart

As can be seen, the VaR trajectory calculated in this article encircles the return sequence like an envelope, with actual returns only occasionally surpassing VaR values. VaR levels peaked in phases during the 2020 global COVID-19 outbreak and the 2022 Russia-Ukraine conflict. This suggests that the EGARCH-T model used in this research yields reliable estimation results and measures crude oil futures market risks precisely.

5. Recommendations and Conclusions

5.1 Conclusion

1,403 daily closing price data points from Shanghai INE crude oil futures are used in this study. It carries out an empirical study on Value at Risk (VaR) estimation for crude oil futures market risk using the GARCH framework by creating a GARCH model. The results show that the RETURN sequence is a stationary time series with a non-normal distribution and a left-skewed, leptokurtic distribution pattern; the returns on INE crude oil futures show non-normality with leptokurtic characteristics. There is some autocorrelation in the return sequence of INE crude oil futures, although it is not statistically significant. Additionally, there is a "leverage effect" in the price changes of crude oil futures, which means that negative information might cause bigger market moves than positive information of the same scale. The highest possible loss for INE crude oil futures at a 95% confidence level is 4.634% of the asset's market value, which suggests a rather high level of risk. Additional investigation reveals that the EGARCH-T model's Value at Risk (VaR) assessment produces acceptable outcomes and efficiently gauges hazards in the crude oil futures market.

5.2 Recommendations

In the area of financial risk management, empirical research on VaR value estimation for crude oil futures market risks based on GARCH models is an important topic. Several suggestions for this study are as follows: First, the features of the crude oil futures market, including price volatility and market efficiency, should be thoroughly taken into account while choosing GARCH models. To guarantee the accuracy of VaR estimation, the best GARCH model for characterizing crude oil futures market risk should be chosen by contrasting various models (e.g., GARCH(1,1), EGARCH, and TGARCH). Second, the choice of holding periods and confidence levels should be the main emphasis of the VaR value computation. VaR values at different confidence levels and holding periods reflect the risk tolerance of investors with varying risk preferences. In order to give thorough risk measuring data, the study should include a variety of confidence levels and holding periods. Additionally, the model should undergo robustness testing to guarantee the accuracy of VaR estimates. This includes tests to determine whether the model accurately incorporates market risk, such as residual testing and parameter stability checks. Simultaneously, additional risk evaluation methods (such as ES, CVaR, etc.) can be compared with VaR data to validate the effectiveness of VaR estimation. Lastly, the connection effects between the crude oil futures market and other financial markets should be the main focus of empirical investigation. Risks may spread throughout the crude oil futures market due to its strong linkages with other financial sectors, including the stock and bond markets. Consequently, the study shall evaluate the sources and transmission mechanisms of risks in the crude oil futures market, providing decision support for cross-market risk management and asset allocation.

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