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Solving Decision Making Problems in Production Processes Based on Bayesian Approach and Snake Optimization Algorithm

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Abstract: In this paper, we study the decision-making problem in the production process to maximize the net profit by using the unit net profit optimization model and the discrete snake optimization algorithm. Firstly, a sampling and testing scheme based on Bayesian method is proposed to design the prior distribution, update the posterior distribution and determine the termination conditions to output the minimum number of tests. Secondly, a two-layer optimization model of unit net profit is constructed to comprehensively consider the cost and profit of each link in production. Finally, the discrete snake optimization algorithm is used to solve the model, and the optimal solution is approximated through specific operation steps to obtain the best decision-making scheme and net profit, which provides a reference for production decision-making.

1. Introduction

In the production process, how to make optimal decisions to maximize net profit is a key issue for enterprises. Traditional decision-making methods are often difficult to consider the complexity and uncertainty of the various aspects of production [1]. This paper aims to solve this problem by Bayesian method and snake optimization algorithm and constructs a two-level optimization model of unit net profit, which realizes multilevel decision optimization more efficiently.

Firstly, the sampling inspection based on Bayesian method is applied to design the prior distribution, update the posterior distribution and determine the termination conditions, to output the minimum number of inspections and accurately control the quality of parts while reducing the inspection cost [2]. Next, a two-layer optimization model of unit net profit is constructed, with the lower planning aiming to minimize the total cost of producing semi-finished products, and the upper planning pursuing the maximum net profit, which is a comprehensive consideration of cost and profit from spare parts to finished products [3]. Finally, the discrete snake optimization algorithm is used to solve the model, breaking through the limitations of traditional algorithms to quickly find the optimal solution. Through these methods, it provides a scientific basis for enterprises in the production decision-making and helps to improve economic efficiency.

2. Sampling detection and output based on Bayesian approach

2.1 Optimized sampling detection based on Bayesian approach

In this paper, a dynamic sampling detection method based on Bayesian method to optimize the number of detections is proposed. The detection program takes "draw one test one" as the basic sampling detection strategy, firstly, according to the nominal value given in the title, design the Beta prior distribution, update the posterior distribution according to the actual sampling situation, and terminate the sampling when the nominal value is lower (higher) than the lower limit (upper limit) of the confidence region, plot the curve equation about the number of samples and the number of defective products, construct the rejection domain (reception domain), and stop the sampling detection when the number of defective products falls into the rejection domain (reception domain). Construct the rejection domain (reception domain) and stop sampling when the number of detected defective products falls into the rejection domain (reception domain), make the choice of rejecting (receiving) the batch of parts and components, and output the minimum number of inspections [4].

2.1.1 A priori distributions

This study designs a sampling scheme through a quality inspection case. The supplier claims that the nominal value of the parts is 0.1. Based on the meaning of the nominal value, consider that 0.1 is the upper limit of the defective rate of the parts supplied by the supplier, i.e., the defective rate of the parts is concentrated in the interval of $[0,0.1]$. The Beta distribution is chosen as the prior distribution Prior(p):

$$p \sim \mathrm{Beta}(\alpha, \beta), \tag{1}$$

Where, to design a prior experience that is more in line with the nominal value, let $P(0.1 \leq p \leq 1) = 0.05$, the purpose is to let the inferiority rate in the Beta prior distribution fall into the interval $[0,0.1]$ with a 95% probability. This condition is satisfied when $\alpha = 0.1$ and $\beta = 6$. And the mean value of Beta distribution meets the actual situation of lower than the upper limit of defective rate, currently, the Beta distribution is shown in Figure 1.

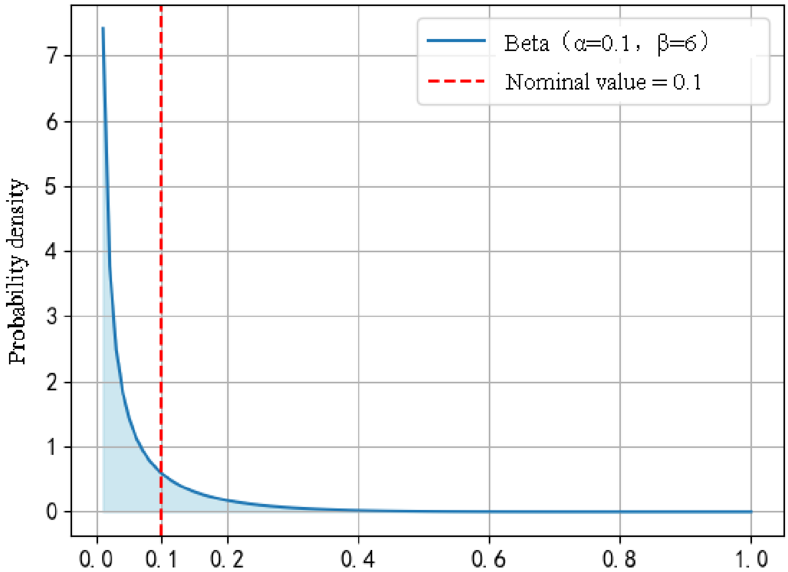

Figure 1. Beta a priori distribution plot

2.1.2 Updating the posterior distribution

After each sampling, new data is obtained. Let we sequentially test a total of n samples, among which there are x inferior products. Then the likelihood function after testing can be expressed as:

$$L(p \mid x,n) = p^x(1-p)^{n-x} \tag{2}$$

According to Bayes' law, the posterior distribution $Posterior(p|x,n)$ is positively related to:

$$\text{Posterior}(p \mid x,n) \propto L(p \mid x,n) \cdot \text{Prior}(p) \tag{3}$$

For the Beta-distributed prior, the updated posterior is still the Beta-distribution with the following parameter updates:

$$p \mid x,n \sim \text{Beta}\left(\alpha_{post}, \beta_{post}\right) \tag{4}$$

2.1.3 Termination conditions

The mathematical expression for the lower limit of the 95% confidence interval of the posterior distribution is:

$$P(p > p_0) = 1 - \text{BetaCDF}\left(p_0 \mid \alpha_{post}, \beta_{\text{post}}\right) \tag{5}$$

Require the following conditions to be met to terminate the sample and reject it:

$$\text{BetaCDF}\left(p_0 \mid \alpha_{\text{post}}, \beta_{\text{post}}\right) \leq 0.05 \tag{6}$$

That is, the lower confidence limit of the posterior distribution is greater than the nominal value, at this time, it is considered that the rate of defective products has exceeded the nominal value of 0.1, and the batch can be rejected.

2.2 Sampling and testing program output

2.2.1 Sampling and testing ideas

Dynamic sampling and testing program that after sampling n times, found that the number of defective products for the first time is greater than x, immediately terminate the sampling and testing, and reject the batch of parts. Based on this strategy can be calculated based on the posterior distribution of the critical equation expression for the maximum number of samplings as:

$$\int_0^{p_0} \frac{t^{\alpha_0+x-1}(1-t)^{\beta_0+n-x-1}}{B(\alpha_0+x, \beta_0+n-x)} dt = 0.05 \tag{7}$$

The equation is a curvilinear equation for the number of samples n versus the detection x. The condition is to let the nominal value be less than the lower limit of the confidence domain, and the curve is a critical curve where the nominal value coincides with the lower limit.

2.2.2 Analysis of results

As can be seen from Figure 2, when the number of sampling times gradually increases, each increase in the rejection condition of the number of substandard products requires more sampling tests, and the rejection domain gradually spreads. This sampling method designed in this paper and China's General Administration of Quality Supervision, Inspection and Quarantine is now the implementation of the GB/T 2828.1-2012 No [5].

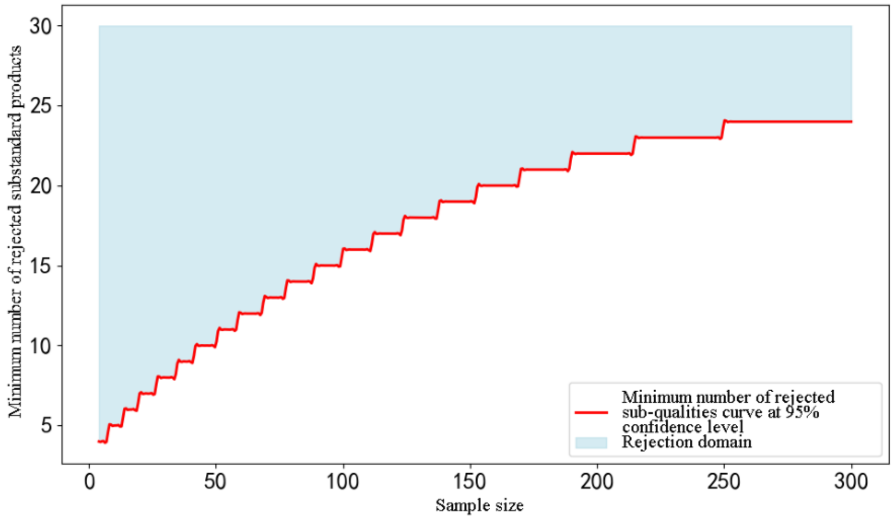

Figure 2. Rejection domain of Case 1 detection scheme

3. Establishment and Solution of Unit Net Profit Optimization Model

3.1 Establishment of unit net profit optimization model

In this section, to solve the problem of making the decision to maximize the net profit, the unit net profit optimization model is proposed, with 0-1 variables δ_i, ζ, γ as the decision variables, and the net profit Profit as the objective function, and the following optimization model is finally summarized:

$$\max \text{Profit } = \max\{n_0 \times w - (c_1 + c_2) - (\delta_1 \times t_1 + \delta_2 \times t_2 + \zeta \times n \times t_3) \tag{8}$$

$$\text{s.t.} \begin{cases} \delta_i \in \{0,1\}, i = 1,2 \\ \zeta \in \{0,1\}, \\ \gamma \in \{0,1\}, \\ n = \max\{\delta_1(1-p_1) + (1-\delta_1), \delta_2(1-p_2) + (1-\delta_2)\}, \\ n_0 = \zeta \times \delta_1 \times \delta_2 \times (1-p_c) \times n + \zeta \times \delta_1 \times (1-\delta_2) \times (1-p_2) \times (1-p_c) \\ \ +\zeta \times (1-\delta_1) \times \delta_2 \times (1-p_1) \times (1-p_c) \\ \ +\zeta \times (1-\delta_1) \times (1-\delta_2) \times (1-p_1) \times (1-p_2) \times (1-p_c) + (1-\zeta) \times n, \\ n_1 = (1-\zeta) \times \delta_1 \times \delta_2 \times p_c \times n + (1-\zeta) \times \delta_1 \times (1-\delta_2) \times (p_2 + (1-p_2)p_c) \\ \ +(1-\zeta) \times (1-\delta_1) \times \delta_2 \times (p_1 + (1-p_1)p_c) \\ \ +(1-\zeta) \times (1-\delta_1) \times (1-\delta_2)(1-(1-p_1)(1-p_2) + (1-p_1)(1-p_2)p_c), \\ n_2 = \zeta \times \delta_1 \times \delta_2 \times p_c \times n + \zeta \times \delta_1 \times (1-\delta_2) \times (p_2 + (1-p_2)p_c) \\ \ +\zeta \times (1-\delta_1) \times \delta_2 \times (p_1 + (1-p_1)p_c) \\ \ +\zeta \times (1-\delta_1) \times (1-\delta_2)(1-(1-p_1)(1-p_2) + (1-p_1)(1-p_2)p_c) \end{cases} \tag{9}$$

In this model, the decision variables are four 0-1 variables, and all the remaining parameters are known quantities given in the problem, which ultimately results in the optimization model of net profit per unit.

3.2 Model Solution

If part 1 fails and part 2 passes, resulting in a nonconforming finished product, the cost w_0 in the disassembly cycle can be:

$$w_0 = p_c^{1-1}(1-p_1)\left(y+\frac{c_1}{1-p_1}+c_3\right)+p_c^{2-1}(1-p_1)\left(2y+\frac{c_1}{1-p_1}+2c_3\right) \tag{10}$$

The above computation and judgment approach reduces the time complexity of the computation and removes the aspect of the number of decision splits. The problem is simplified to 8 cases and iterated through all the cases to find the decision scheme that optimizes the net profit in each case [6].

The histogram of the net profit under the 8 decision cases is shown in Figure 3, and the net profit of Decision 6 and Decision 8 is above 20, which is a higher level in the graph, to select a more optimal processing scheme.

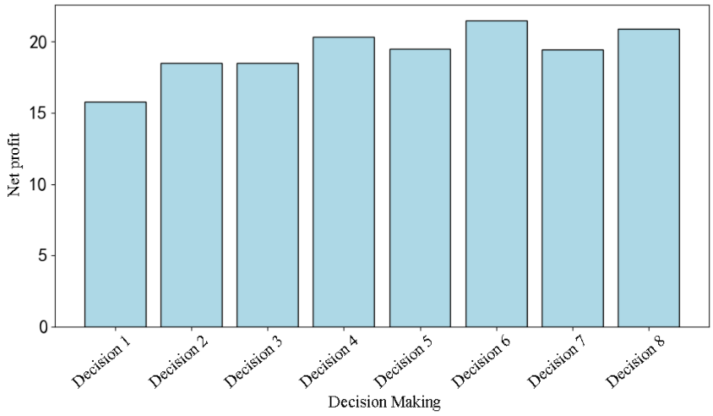

Figure 3. Histogram of net profit under different decisions

4. Establishment of two-layer Optimization Model of Unit Net Profit and Discrete Snake Optimization

4.1 Two-tier optimization model of unit net profit

4.1.1 Construction of the lower layer planning model

The lower-level planning model is related to the process of assembling spare parts into semi-finished products.

The quantity assembled into semi-finished product 1 is denoted as $N1$:

$$N_1 = \max\{\delta_1(1-p_1)+(1-\delta_1), \delta_2(1-p_2)+(1-\delta_2), \delta_3(1-p_3)+(1-\delta_3)\} \tag{11}$$

Calculate the cost of each item, where the purchase cost is:

$$W_{h11} = c_1 + c_2 + c_3 \tag{12}$$

Testing cost is:

$$W_{h12} = \delta_1 t_1 + \delta_2 t_2 + \delta_3 t_3 + \omega_1 n_1 t_{h1} \tag{13}$$

Assembly cost is:

$$W_{h13} = n_1 c_{h1} \tag{14}$$

Disassembly cost is:

$$W_{h14} = \eta_1(N_1 - N_1')y_{h_1} \tag{15}$$

The total cost W_{h10} for Subplan 1 is:

$$W_{h10} = W_{h11} + W_{h12} + W_{h13} + W_{h14} \tag{16}$$

The optimization objective summaries are summed up to obtain the lower-level planning model for this unit net profit two-tier planning model as:

$$\begin{aligned}\min W_{h0} &= \min(W_{h10} + W_{h20} + W_{h30}) \\ &= \min\{(c_1 + c_2 + c_3) + (\delta_1 t_1 + \delta_2 t_2 + \delta_3 t_3 + \omega_1 n_1 t_{h1}) + n_1 c_{h1} + [\eta_1 (N_1 - N_1') y_{h_1}] \\ &+ (c_4 + c_5 + c_6) + (\delta_4 t_4 + \delta_5 t_5 + \delta_5 t_5 + \omega_2 n_2 t_{h2}) + n_2 c_{h2} + [\eta_2 (N_2 - N_2') y_{h_2}] \\ &\text{s.t.} \begin{cases} \delta_i \in \{0,1\}, i = 1,2,\dots,8 \\ \omega_j \in \{0,1\}, j = 1,2,3 \\ \eta_j \in \{0,1\}, j = 1,2,3 \end{cases}\end{aligned} \tag{17}$$

In this lower-level planning model, the optimization objectives of the three sub-programs are expressed separately and finally summed up to obtain the lower-level planning model with $\delta_i, \omega_j, \eta_j$ as 0-1 decision variables and the optimization objective of minimizing the total cost of producing semi-finished products.

4.1.2 Construction of upper-level planning model

The upper-level planning model takes the maximum net profit as the optimization objective and describes the impact of decisions on net profit through 0-1 decision variables. The steps are as follows:

(1) Calculate the number of finished products N:

$$N = \max\{N_1', N_2', N_3'\} \tag{18}$$

(2) The number of finished products put on the market is N_0:

$$N_0 = f(i,j,k,\zeta) \tag{19}$$

(3) The number of unqualified finished products is $N - N_0$:

$$N - N_0 = \max\{N_1', N_2', N_3'\} - f(i,j,k,\zeta) \tag{20}$$

(4) Determine the number of defective products, the number of unqualified products returned by the user is recorded as $g(i,j,k,\zeta)$, the total number of defective products is recorded as $h(i,j,k)$, and then:

$$\max\{N_1', N_2', N_3'\} - f(i,j,k,\zeta) + g(i,j,k,\zeta) = h(i,j,k) \tag{21}$$

(5) Calculate each cost and profit. According to the target net profit function of the upper-level planning model is net profit = total profit - (testing cost + assembly cost + return cost), expressing the net profit Profit' as the target function, and then:

$$\text{Profit'} = W_0 - (W_{c1} + W_{c2} + W_{c3} + W_{c4}) \tag{22}$$

Based on this relation, the upper-level planning model is found to be:

$$\begin{aligned}\max \text{Profit} &= \max[W_0 - (W_{c1} + W_{c2} + W_{c3} + W_{c4})] \\ &= \max\{f(i,j,k,\zeta)w - [\zeta \max\{N_1', N_2', N_3'\}t + \max\{N_1', N_2', N_3'\}c \\ &+ \gamma h(i,j,k)y + (m+w)g(i,j,k,\zeta)]\} \text{ s.t.} \begin{cases} \gamma \in \{0,1\}, \\ \zeta \in \{0,1\} \end{cases}\end{aligned} \tag{23}$$

According to this equation, can then find out the relationship between the optimization objective of the upper planning model and the distribution law of the decision variables, and finally combine with the lower planning model to summarize the unit net profit two-tier planning model.

4.2 Discrete snake optimization algorithm to solve the model

To solve the improved model, this study adopts the discrete snake optimization algorithm (Serpent Optimization Algorithm (SO)), which is inspired by the mating behavior of snakes and has the advantages of simple calculation and high efficiency [6].

In the snake optimization algorithm, the initialization process is denoted as:

$$y_i = lb + r \times (ub - lb) \tag{24}$$

Where lb and ub are equal to 0 and 1, respectively; "r" is a random number in $[0,1]$; and the population consists of males and females, assuming 50% males and 50% females.

Individual coding is set to $[x_1, x_2, \dots, x_{17}]$ and each value is a 0-1 variable.

The negative value of total profit is taken as the fitness function to simplify the two-layer planning model and prioritize the upper layer objective function. Starting from the lower-level spare parts detection decision, gradually go to the upper level semi-finished and finished product decision, and then adjust the lower-level decision according to the upper-level fitness. When dealing with the quantity mismatch problem, follow the principle of under-compensation and add the expected cost of genuine goods.

Combining the worst and best male and female individuals in the solution space search, Eq:

$$\begin{cases} y_{i,m}(g+1) = y_{i,m}(g) + r_1 \times \left(y_{\text{best},m} - y_{i,m}(g)\right) - r_2 \times \left(y_{\text{worst},m} - y_{i,m}(g)\right) \\ y_{i,f}(g+1) = y_{i,f}(g) + r_1 \times \left(y_{\text{best},f} - y_{i,f}(g)\right) - r_2 \times \left(y_{\text{worst},f} - y_{i,f}(g)\right) \end{cases} \tag{25}$$

Where $y_{\text{best},m}$ and $y_{\text{worst},m}$ are the best and worst males in the population, and $y_{\text{best},f}$ and $y_{\text{worst},f}$ are the best and worst females in the population, respectively; and r_1 and r_2 are random numbers in the interval $(0,1)$, which can speed up convergence.

Improve the quality of the solution by eliminating the low fitness decision group and introduce the mutation operation:

$$O_i(g+1) = \begin{cases} y_{\text{best}}(g) + \text{sign}(r - 0.5) \times [lb + r \times (ub - lb)] & \text{, if rand } < 0.5 \\ y_i(g) + \text{sign}(r - 0.5) \times [lb + r \times (ub - lb)] & \text{, else if} \end{cases} \tag{26}$$

$$y_i(g+1) = \begin{cases} O_i(g+1) & \text{, if } F(O_i(g+1)) < F(y_i(g+1)) \\ lb + r \times (ub - lb) & \text{, else if} \end{cases} \tag{27}$$

Where $i = 1,2, \dots, N/2, O_i$ is the new position randomly searched around the historical individuals.

For the first 50% of individuals variant and greedy operation:

$$My_i(g) = y_{p1}(g) + F \cdot \left(y_{p2}(g) - y_{p3}(g)\right) \tag{28}$$

Where $My_i(g)$ is the individual of the population after the greedy operation; $y_{pi}(i = 1,2,3)$ is the individual within the original population. F is the scale factor:

$$F = \frac{1}{2}\left(\sin\left(2\pi \times f_{req} \times g\right) \times (g + G) + 1\right) \tag{29}$$

4.3 Analysis of results

The net profit iteration curve as Figure 4:

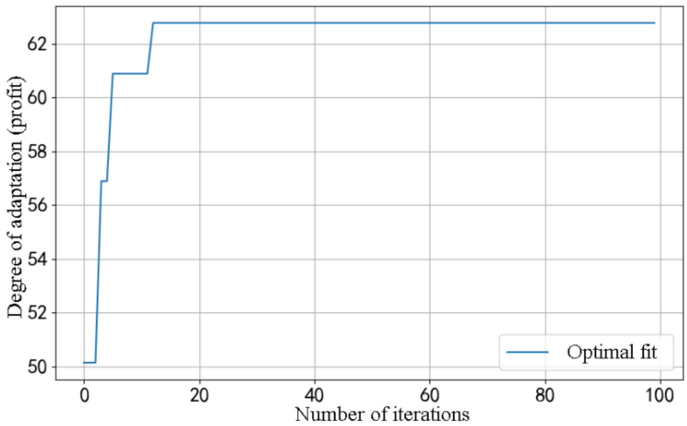

Figure 4. Net profit iteration curve

For the Figure 4, the objective function tends to converge at 24 iterations, when the maximum value of the fitness (i.e., net profit) is 62.78, i.e., the net profit of selling a finished product is 62.78%.

The corresponding decision-making situation is that, for parts, all parts are not tested except for part 2; for semi-finished products, all semi-finished products are tested, but not disassembled; for finished products, all costs are tested and disassembled at the same time. At this point, problem three is solved[7].

5. Conclusions

This paper focuses on the decision-making problem in the production process, integrates the use of Bayesian method and discrete snake optimization algorithm, and constructs the optimization model of modal unit net profit and solves it in order to maximize the profit in production.

Firstly, the sampling test based on Bayesian method can dynamically adjust the number of sampling tests by designing the prior distribution, updating the posterior distribution and setting the termination conditions, outputting the minimum number of tests and consistent with the trend of changes in the national standard, which reduces the cost of testing while ensuring the closeness to the reality and feasibility.

Secondly, a two-layer optimization model of unit net profit is constructed, with the lower planning aiming at minimizing the total cost of producing semi-finished products, and the upper planning pursuing the maximum net profit, which comprehensively considers the cost and profit of each production link and provides a systematic framework for decision-making.

Finally, the discrete snake optimization algorithm is used to solve the improved model, which can quickly approximate the optimal solution in the complex solution space through specific initialization, search, mutation and other operations. After the experiment, the algorithm tends to converge at 24 iterations, obtains the maximum net profit of 62.78%, and gives a specific decision-making scheme, which provides a scientific basis for production decision-making.

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