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Design of Performance Allocation Schemes Based on TOPSIS Analysis and Linear Programming Models

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Abstract: The purpose of this paper is to design a performance allocation scheme for researchers based on TOPSIS analysis and linear programming model. First, for the factors affecting performance, this paper transforms qualitative data into quantitative data by coding job titles and detecting outliers and missing values. Then, the TOPSIS method is used to calculate the weights of the indicators through SPSS software, and the performance of researchers is ranked according to the comprehensive score, and then the bonuses are allocated based on the principle of linear allocation. Secondly, this paper constructs a linear programming model at the team level, calculates the total team score and further allocates bonuses based on performance weights. Finally, for the missing data and distribution, the K-S test is used to test normality, and on this basis, the data are adjusted to ensure the accuracy and validity of the research results. The performance allocation scheme proposed in this paper can realize the fair distribution of rewards among researchers and provides a quantitative and systematic decision-making tool for research management.

1. Introduction

In the current context of rapid development of science and technology, the performance evaluation of scientific researchers [1] is particularly important. The purpose of this paper is to explore the optimization scheme of scientific research personnel performance allocation through the comprehensive use of a variety of computer-related algorithms and models. The study adopts the TOPSIS analysis method [2], which is a comprehensive evaluation of scientific research results to ensure the scientificity and fairness of performance allocation. Meanwhile, the linear programming model [3] was used to rationally allocate performance bonuses and maximize the performance scores of teams and individuals [4]. In addition, K-S test [5] was used to verify the effectiveness of the performance allocation scheme and ensure the reliability of the results.

The study covers the classification and scoring of scientific research achievements, the incentive mechanism of teamwork [6], and the expansion strategy of career income. By systematically analyzing the performance of 20 research post employees and combining various types of factors such as title, length of service and quality of results, a reasonable performance allocation scheme is formulated. Ultimately, the research in this paper not only provides scientific basis for scientific research institutions, but also provides an effective reference for the construction of incentive mechanism for scientific and technological personnel, which promotes the transformation of scientific and technological achievements and the improvement of innovation system. Through these methods, this paper shows how to realize the dual goals of fairness and incentive in performance evaluation, which has important practical significance and application value.

2. Bonus Weighting Scheme

2.1 Data Preprocessing

This section preprocesses the information related to the research achievements of 20 research post employees of a college for the year 2023, such as checking for outliers, missing values, and duplicated data, in order to prevent any impact on the accuracy and credibility of the calculation results.

(1) First, it is necessary to check whether there are outliers or missing values in the data set. For missing values use MATLAB's find function to find the missing values in the data set and find that there are no missing values in the data. For outliers processing, through the analysis, the number of graduate students under the tutor of serial number 17 is 10 for the reasonableness analysis, and the box plot in SPSS software is used to react to the centralized and discrete trends of the distribution of the continuous type of quantitative data and to identify and process them.

(2) Analyzing the data found that there are qualitative variables affecting the weight calculation, quantitative evaluation of the title into quantitative variables for coding and quantification and entry and analysis. To facilitate data management and analysis, a mapping relationship between job titles and codes was established to associate each job title with the corresponding code. The codes were coded as 4, 3, 2, and 1 for full senior, associate senior, intermediate, and junior, respectively.

2.2 Distribution Modeling

For the scientific research achievements of employees in research positions, the establishment of the distance between superiority and inferiority solution (TOPSIS) model will be analyzed and processed by SPSS.

(1) Normalization of Forwarding Matrix

Assuming that there are m objects to be evaluated, n evaluation indicators (already for forwarding) constitute the forwarding matrix as follows ($m = 20,\ n = 17$):

$$X = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \tag{1}$$

Then the matrix after its normalization is denoted as S. Each element of S (where x_{ij} denotes the jth indicator for the ith person; a_{ij} denotes the item corresponding to each person ($i = 1,2.\dots,20$) ($j = 1,2,\dots.15$):

$$s_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}} \tag{2}$$

That is, the normalization matrix z obtained by (each element / sum of squares of the elements of the columns located under the root sign):

$$z = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1m} \\ z_{21} & z_{22} & \cdots & z_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nm} \end{bmatrix} \tag{3}$$

(2) Calculate the Score and Normalize

Define the maximum value:

$$z^{+} = \left(\max\left\{z_{11}, z_{21}, \cdots, z_{n1}\right\}, \max\left\{z_{12}, z_{22}, \cdots, z_{n2}\right\} \cdots, \max\left\{z_{1m}, z_{2m}, \cdots, z_{nm}\right\}\right) \tag{4}$$

Define the minimum value:

$$z^{+} = (\min\{z_{11}, z_{21}, \cdots, z_{n1}\}, \min\{z_{12}, z_{22}, \cdots, z_{n2}\} \cdots, \mathrm{m}in\{z_{1m}, z_{2m}, \cdots, z_{nm}\}) \tag{5}$$

Define the ith ($i = 1,2.\ldots, n$) distance of the evaluation object from the maximum value:

$$D_{\mathrm{i}}^{+} = \sqrt{\sum_{\mathrm{j}=1}^{\mathrm{m}} (Z_{\mathrm{j}}^{+} - Z_{\mathrm{ij}})^2} \tag{6}$$

Define the distance of the ith ($i = 1, 2, \ldots n$) evaluation object from the minimum value:

$$D_{\mathrm{i}}^{-} = \sqrt{\sum_{\mathrm{j}=1}^{\mathrm{m}} (Z_{\mathrm{j}}^{-} - Z_{\mathrm{ij}})^2} \tag{7}$$

Then, the score for which the $i(i = 1,2, \ldots n)$th evaluation object is normalized can be calculated as.

$$S = \frac{D_{\mathrm{i}}^{-}}{D_{\mathrm{i}}^{+} + D_{\mathrm{i}}^{-}} \tag{8}$$

It is obvious that $0 \leq Si \leq 1$, and the larger Si , D_{i}^{+} the smaller, i.e., the closer to the maximum value.

(3) Comparative Processing of Data

By carefully observing the distance between the evaluation object and the optimal solution and the worst solution, it can be found that, according to the ordering, the index value of the index positive high 1 comprehensive score index is the highest, and the primary 8 comprehensive score index is the lowest.

The weight ratio of each index w_{ij} :

$$w_{ij} = \frac{d_j}{\sum_{j=1}^{m} d_j}, \ j = 1, 2 \ldots 20 \tag{9}$$

Calculate the weight of the ith sample value under the jth index to the total sample p_{ij} for:

$$p_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}} \tag{10}$$

2.3 Weight Assignment Model Solving

(1) Weight of Each Factor

After processing the data, it is analyzed that national scientific and technological awards have the highest weight share among all the factors considered, while the weight share of SCI is at the lowest level. In addition, the weights of the remaining factors are relatively close to each other, with no obvious gap. This suggests that national-level scientific and technological awards are given higher importance in the assessment, while SCI has relatively less influence, and the other factors have similar weight status. Such weight allocation may reflect the relative importance attached to different factors in a particular assessment criterion or decision-making process.

(2) Calculation Results of TOPSIS Evaluation Method

The weights are set using SPSS combined with the entropy weighting method and the merit solution distance method is applied to the project. Evaluate and compare the projects using the superiority-distance method. The entropy weighting method was first used to assign weights to each indicator to determine their relative importance in the comprehensive evaluation. The entropy weight method calculates the information entropy of each indicator and determines the weights according to the degree of dispersion of the indicators, thus avoiding the influence of subjective factors on the allocation of weights. On the basis of setting the weights, the superiority solution distance method is used to evaluate the project. The results are obtained as in Table 1:

Table.1. Sorting of Comprehensive Score Index (Partial Data)

Index value	Positive ideal solution distance (D^+)	Negative ideal solution distance (D^-)	Composite score index	Ordering
Positive 1	0.58195822	0.70636396	0.54828207	1
Positive 5	0.60461409	0.62904701	0.50990261	2
⋮	⋮	⋮	⋮	⋮
Primary 8	0.94105002	0.22947928	0.19604744	15

(3) Distribution of Bonus

According to the principle of linear distribution (that is, directly based on the proportion of the weight of each factor), the distribution of bonuses for 20 scientific research workers is obtained, as shown in Figure 1 below

It can be clearly seen from Figure 1 that the first scientific research employee receives the highest amount of bonus. To verify the reasonableness of this result, further detailed comparative analysis of the scientific research achievements of the first employee and other employees was carried out. The results show that the first employee does perform well in terms of the quantity, quality and innovativeness of scientific research achievements, which is in line with the results of the bonus distribution. This shows that the linear distribution method can, to a certain extent, accurately reflect the scientific research contributions of the employees, and has a certain degree of rationality and fairness.

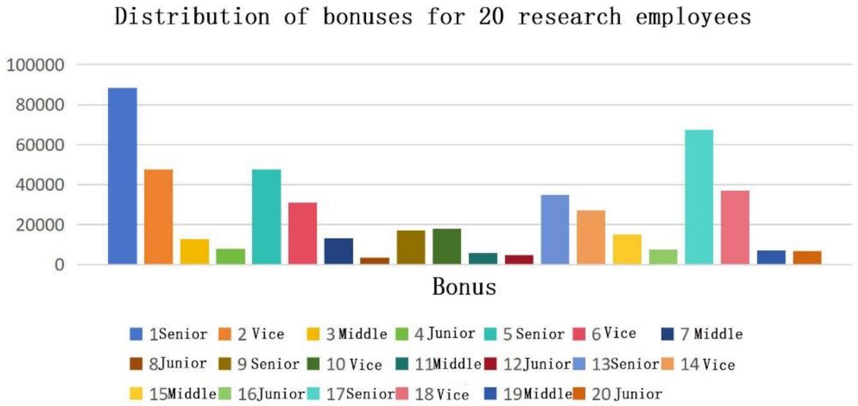

Figure 1. Bar chart of bonus distribution for 20 research employees

3. Analysis of Distribution by Member

3.1 Data Preprocessing

The indicators are categorized and pre-processed with SPSS, first applying the K S one-sample test to put forward the hypothesis of whether the item obeys the normal or uniform or exponential distribution law, and the specific results are shown in Table 2:

Table.2. Test Results

Variable name	Sample size	Upper quartile	Average value	Standard deviation	Skewness	Kurtosis	S-W test	K-S test
Patents	20	0.5	0.55	0.605	0.583	-0.459	0.737(0.000***)	0.318(0.027**)
Award category	20	0	0.05	0.224	4.472	20	0.236(0.000***)	0.538(0.000***)
Literature	20	1	0.75	0.851	1.104	1.067	0.792(0.001***)	0.261(0.109)
Project type	20	1	0.85	0.875	0.315	-1.667	0.766(0.000***)	0.284(0.064*)
Resource management	20	1	1.35	1.348	0.998	1.195	0.855(0.007***)	0.192(0.404)
Cost category	20	6.5	17.95	22.101	1.555	1.942	0.79(0.001***)	0.24(0.170)

Note: ∗∗∗, ∗∗, and ∗ represent 1%, 5%, and 10% significance levels, respectively.

Observing the asymptotic significance of the indicators in Table 2 under normal, uniform and exponential parameters, it is found that all obey the law of normal distribution.

3.2 Establishment of Offline Planning Model

Maximize team scores by establishing linear programming in the optimization model.

(1) Decision Variables

x_{ij} : the number of j results submitted by the ith team.

E: the performance score of the team.

y_i : the amount of performance allocation for the ith team.

(2) Objective Function

The objective of this section is to maximize the total performance reward, i.e., $MaxE$. The objective function can be set as:

$$MaxE = \sum_{i=1}^{20} hx_{ij} \tag{11}$$

Where h represents the weights and E is the performance scores of the four teams.

Maximize the total performance allocation result, i.e.

(3) Constraints

To ensure that the total number of results submitted by each team does not exceed 20, the following constraints are set:

$$\sum_{j=1}^{17} x_{ij} \le 20(i=1,2,3,4;\ j=1,2,\cdots 17) \tag{12}$$

Where i represents teams, j represents projects, and x_{ij} : the number of j outcomes submitted by the ith team.

Similarly, the number of submissions for papers, projects, and lateral arrivals funding should all be non-negative integers, so there:

$$x_{ij} \ge 0 \tag{13}$$

The total award pool is 1 million, i.e.:

$$\sum_{i=1}^{4} y_i \le 100 \tag{14}$$

Where y represents the award pool amount for the team.

Distribute the awards according to the performance-based award program, with the total amount equal to the total award pool multiplied by the team's allocation percentage, i.e.:

$$y_i = \omega \times 100 \tag{15}$$

Where y_i represents the total amount of the award, and ω represents the team's share of the total pool.

The final model is:

$$\text{Max}\, E = \sum_{i=1}^{20} hx_{ij} \tag{16}$$

$$s.t \begin{cases} \sum_{j=1}^{17} x_{ij} \le 20(\text{i}=1,2,3,4;\ j=1,2,\cdots 17) \\ x_{ij} \ge 0 \\ \sum_{i=1}^{4} y_i \le 100 \\ y_i = \omega \times 100 \end{cases} \tag{17}$$

3.3 Optimization Model Solving

This section uses a linear programming approach to solve the model. By transforming the objective function and constraints into a system of linear equations, and using MATLAB software programming to perform calculations, the total scores of the 4 teams are obtained as shown in Figure 2 below.

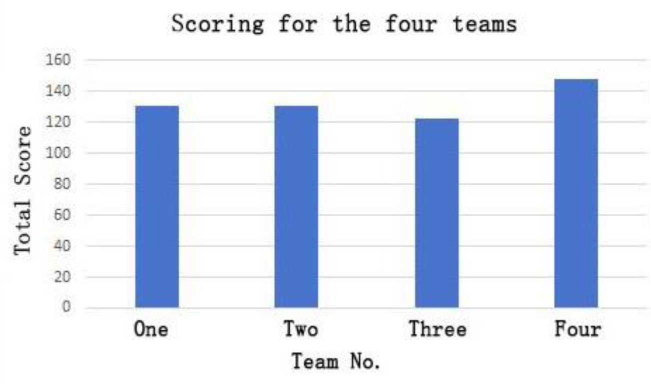

Figure 2. Histogram of the total scores of the 4 teams

The allocation of the teams after the ranking is done according to the allocation ratio, obtaining the following Figure 3.

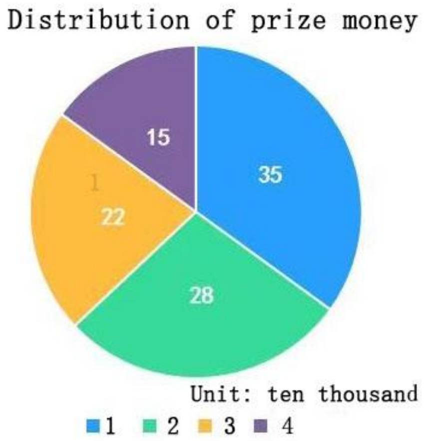

Figure 3. Pie chart of prize distribution

The four teams received 28, 22, 15, and 350 thousand dollars in bonuses respectively.

After the performance bonus is determined at the team level, it needs to be distributed to each member of the team. Using the weight distribution obtained during the solution, the performance allocation ratio of each member of the team is found, in which the performance allocation results of the five members of the No. 1 team can be expressed as in Figure 4.

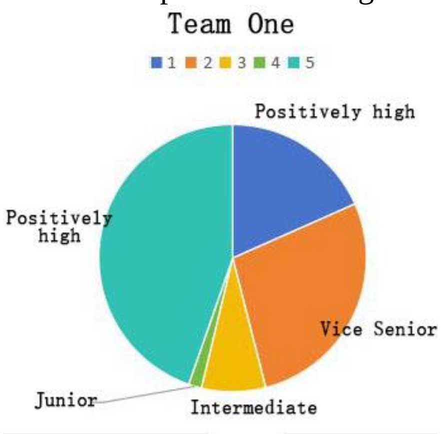

Figure 4. Pie chart of individual performance allocation of team No. 1

According to the model solving results, the specific performance allocation amount of each team can be obtained. In order to ensure internal balance and fairness, the results can be further analyzed and discussed. For example, comparing the performance allocation amounts of different teams, checking whether they are in line with the expected allocation ratios, and analyzing the impact of the weights of different outcome categories on the allocation results.

4. An Exploration of Total Performance Maximization

4.1 Rational Optimization Model

Funding is not considered as part of the research output, so a total of 7 indicators will be included in the discussion. Using the 7 indicators as variables, an objective function is set to represent the maximization of total performance. Then, by defining constraints, such as resource limitations, time limitations, etc., to ensure that the solution results are in line with the actual situation.

(1) Objective Function

$$Z = \max(P_1 + P_2 + P_3 + P_4) \tag{18}$$

Where Z denotes total performance and P denotes level.

(2) Constraints

$$0 \le p_i \le 100000 \quad (\mathrm{i} = 1, 2, 3, 4) \tag{19}$$

The amount of individual performance allocation should correspond to the individual title and allocated funding index:

$$\begin{aligned} p_1 &\le 320000 \times \frac{40000}{40000+25000+15000+10000}, \\ p_2 &\le 200000 \times \frac{25000}{40000+25000+15000+10000}, \\ p_3 &\le 100000 \times \frac{15000}{40000+25000+15000+10000}, \\ p_4 &\le 200000 \times \frac{10000}{40000+25000+15000+10000}, \end{aligned} \tag{20}$$

The total performance of the target team is required to be optimal:

$$A_1 > A_i (\mathrm{i}=1,2,3) \tag{21}$$

Performance sum of everyone in each group:

$$S = \sum_{i=1}^{7} (x_i \times h_i) \tag{22}$$

Performance sum for each group:

$$G = \sum_{j=1}^{5} S_j \tag{23}$$

Where, x denotes the number of each project, h denotes the weight corresponding to the project, A denotes the team, the S denotes the performance sum of individuals in each group, and G denotes the performance sum of each group.

The final model is:

$$Z = \max\left(P_1 + P_2 + P_3 + P_4\right) \tag{24}$$

$$\text{s.t}\begin{cases} P = \dfrac{a}{b} \times c + \dfrac{e}{f} \times d \\ 0 \le p_i \le 100000 \\ A_1 > A_i \\ S = \sum_{i=1}^{7}\left(x_i \times h_i\right) \\ G = \sum_{j=1}^{5} S_j \end{cases} \tag{25}$$

Where a denotes the annual individual arrival funding, b denotes the allocation of funding targets by individual title, c denotes the individual performance base, e denotes the annual individual achievement score, f denotes the score of all scientific and technological achievements, and d denotes the total amount of achievement bonus.

4.2 Rationalization Model Solving

By utilizing SPSS software programming, the weights of each item are obtained as follows in Figure 5.

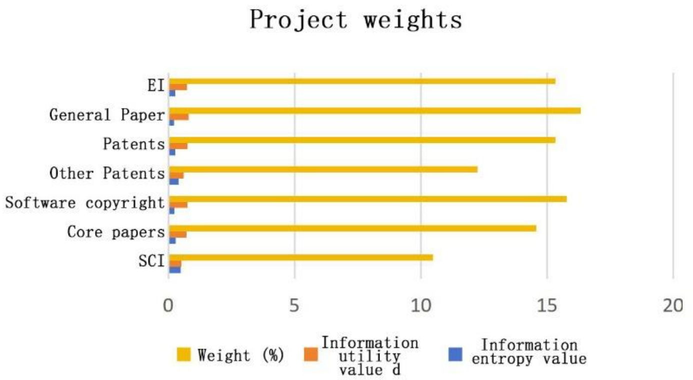

Figure 5. Comparison of item weights

In the process of calculating the weights using the entropy weight method, it is found that the target project ranks third. However, this paper explicitly requires the total performance of the target team to be at the optimal level. In order to meet this requirement, the data are optimized and processed, while giving full consideration to the balance and fairness within the team. The weight distribution of each project is obtained through the calculation of entropy weight method. However, relying on the weights alone does not fully reflect the overall performance of the team. Therefore, it is necessary to incorporate the factors of balance and fairness within the team in data processing.

When optimizing data processing, these factors are considered comprehensively to achieve the optimal total performance of the target team. Through careful evaluation and weighing, the weight allocation can be adjusted to make the team's total performance more in line with the requirements, while maintaining the internal balance and fairness of the team. The following Figure 6 is finally obtained.

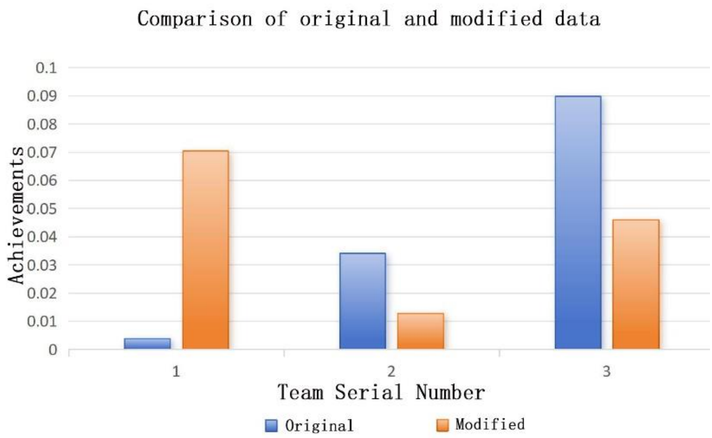

Figure 6. Comparison of original data and optimized data

Firstly, the person with the highest product of title and degree of each team was selected for processing, and the performance of the target team was calculated not to be ranked first among the three teams. The model was processed by taking the person with the smallest product of title and degree for each team, and it was calculated that the utility of the objective function was ranked first among the three teams.

5. Conclusions

The study proposes a research performance evaluation and allocation scheme based on TOPSIS model and linear programming, which combines multi-dimensional factors such as title, research achievements, team contribution, etc., aiming to realize a fair and scientific distribution of rewards. First, the rationality of the evaluation indexes is ensured through data preprocessing and coding, and the TOPSIS model is used to comprehensively assess the performance of researchers and allocate bonuses according to the score ranking. Second, for team performance allocation, K-S test was used to verify the normality of the data, and linear programming was used to optimize the internal allocation of the team, considering the individual contribution and team cooperation. Finally, a reasonable bonus allocation mechanism is designed by combining career income and performance evaluation, which can effectively motivate researchers to innovate and collaborate, and improve the overall performance of research institutions.

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