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Research on the Goldbach Conjecture

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Abstract: This paper is a comprehensive discussion of following three papers: *An Even Number Divisible by 6 could be Expressed as the Sum of Several Groups of Two Prime Numbers*[1], *Research on an Even Number Which Plus 2 Can be Exactly Divided by 6 Can be Expressed as the Sum of Several Sets of Two Prime Numbers*[2], *Research on an Even Number Which Minus 2 Can be Exactly Divided by 6 Can be Expressed as the Sum of Several Sets of Two Prime Numbers*[3], which is published in sections with too long solution. In this paper, the relationship between the number D of even P(1,1) prime number pairs and the number L of twin prime numbers smaller than the even number is synthesized, and the characteristics of the P(1,1) prime number pairs and the characteristics of the group number of prime pairs are analyzed, and the factors affecting the number of P(1,1) prime number pairs are analyzed. The Formulas of P(1,1) prime pairs using the number of twin primes less than the even number is given and the Goldbach Conjecture is proved valid.

1. Composition characteristics of even P(1,1) prime pairs

The primes except 2 and 3 are divided into two categories: $P_A=6m+1$, $P_B=6m-1$[4]($m\geqq 1$).

Even numbers greater than or equal to 10 (represented by N) are divided into three classes: $N_1=6m-2$, $N_2=6m$, $N_3=6m+2$($m\geqq 2$), all of which can be Expressed as the sum of one or more groups of two primes (P(1,1) prime pairs). The P(1,1) prime pair of N_2 is a P_A prime plus a P_B prime[1]. The P(1,1) prime pair of N_1 is two P_B primes [2], and the P(1,1) prime pairs of N_3 are two P_A primes [3]. If odd numbers (N-3) are primes, N_1 and N_3 also have (P+3) prime pairs.

6 and 8 are the sum of two prime numbers with 3.

2. Factors that affect the number of P(1,1) prime pairs

(1) Classification of even numbers: The number of P(1,1) prime pairs of N_2 is about equal to or more than the number of twin primes smaller than the even number (excluding the twin primes (3,5), the same below) [1], the number of P(1,1) prime pairs of N_1, N_3 is about equal to or more than half of the number of twin primes smaller than the even number (rounded, the same below) [2], [3] (see Tables 1,2,3).

(2) The size of even numbers: Based on classification, the general trend is that the larger the N is the more prime pairs there are (see Table 2, 3).

(3) The influence of prime factors: The prime factors mentioned in this paper refer to the P_A and P_B primes less than $\sqrt{N}$ that can divide $\frac{N_2}{6P_A}$, $\frac{N_2}{6P_B}$ [1], $\frac{N_1+2P_A}{6P_A}$, $\frac{N_1-2P_B}{6P_B}$ [2], $\frac{N_3-2P_A}{6P_A}$ or $\frac{N_3+2P_B}{6P_B}$ [3] (even prime factors are not equal to prime factors, 2 and 3 can be prime factors of integers, but not even prime factors, the same prime factors belong to a prime factor), and N factors increase the P(1,1) prime numbers of the even number. The smaller the prime factor is or the more prime factors there are, the more prime pairs there will be (see Table 1,4).

(4) The influence of the starting point of frequency curve intersection [1], [2], [3].

(5) The influence of the intersection of non-composite dotted lines [1], [2], [3].

(6) Whether there is any (P+3) prime pair.

The first three are the main factors of the number of P(1,1) prime numbers, the latter three have obvious effects on the small even number, and the large even number is neglected.

3. Characteristics of P (1, 1) prime number pairs

3.1. The relationship between D, N, and L can be seen from the curve

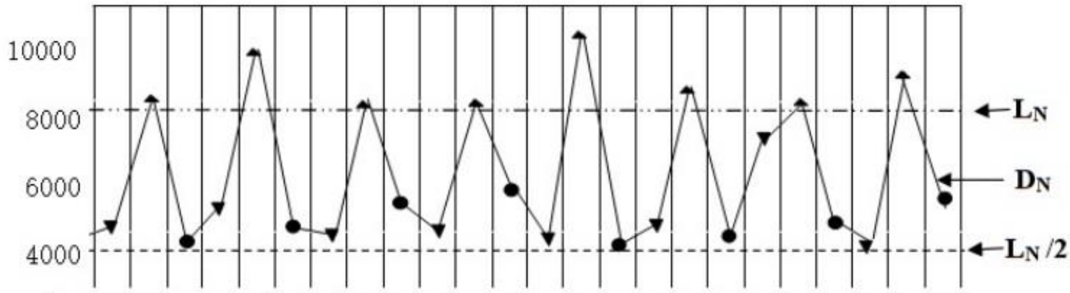
In the plane rectangular coordinate system, the L_N curve is formed by connecting points with N as the horizontal coordinate and the group number of the twin prime number L_N smaller than N as the vertical coordinate and then connecting points with N as the horizontal coordinate and $0.5L_N$ as the vertical coordinate to form the $L_N/2$ curve. Both curves are parabolic forms.

3.1.1. D_N curve

In the same coordinate system, points with N as the horizontal coordinate and N's P(1,1) prime pair number D_N as the vertical coordinate are connected to form a D_N curve (see Figure 1), in which D_{N1}, D_{N2}, and D_{N3} are included. The adjacent coordinate points of the curve can be very different and the standard curve cannot be made, but there are obvious rules in general: D_{N2} is much larger than adjacent D_{N1} and D_{N3}, while the difference between D_{N1} and D_{N3} is small. Combined with the influence of prime factors, D_{N1}, D_{N2}, and D_{N3} of each group with the same even number of m values (N_1=6m-2, N_2=6m, N_3=6m+2) form triangular spines. The D_N curve is composed of numerous such spines, the initial shape is like a crocodile tail. With the increase of even numbers, the shape is like a hand-painted porcupine back. The upper spines are irregular, reaching the L_N curve or higher. The bottom corners of the spines reach the $L_N/2$ curve or above, and many of the lowest points are consistent with the $L_N/2$ curve. The general trend of the D_N curve is to increase with the increase of even number.

3.1.2. D_{N1} curve, D_{N2} curve, D_{N3} curve

If the D_{N1}, D_{N2}, and D_{N3} coordinate points are connected respectively to form the D_{N1} curve, the D_{N2} curve, and the D_{N3} curve (see Figure 2). Each curve is like an irregular wave shape, the adjacent coordinate points can be very different and cannot be made into a standard curve, but there are obvious rules in the general: D_{N1} curve and D_{N3} curve are basically between $L_N/2$ curve and L_N curve, the high points are uneven, while many of the lowest points are consistent with $L_N/2$ curve. The D_{N2} curve is basically above the L_N curve, the high point is also uneven. Many of the lowest points are consistent with the L_N curve. The general trend of the three curves increases with the increase of even numbers.



N		D_N	Prime factors
1000054	N_1	4566	11, 131, 347
1000056	N_2	8113	
1000058	N_3	4062	
1000060	N_1	5573	5, 31
1000062	N_2	9717	7
1000064	N_3	4420	13, 601
1000066	N_1	4121	47
1000068	N_2	8095	
1000070	N_3	5393	5, 97
1000072	N_1	4161	41
1000074	N_2	8075	
1000076	N_3	5796	7, 11, 17, 191
1000078	N_1	4085	673, 743
1000080	N_2	10811	5, 463
1000082	N_3	4032	
1000084	N_1	4293	19
1000086	N_2	8431	23
1000088	N_3	4080	149, 839
1000090	N_1	7105	5, 7, 13, 157
1000092	N_2	8045	
1000094	N_3	4336	29, 43, 401
1000096	N_1	4052	
1000098	N_2	8979	11
1000100	N_3	5421	5, 73, 137

Figure 1: D_N curve for consecutive even numbers (L_N=8169, $0.5L_N$=4085)

3.2. Judge the number of P (1,1) prime pairs from the properties of even numbers

If N can only be divisible by 2 and cannot be divisible by other prime numbers which are greater than 2 and less than $\sqrt{N}$, $D_N \approx 0.5L_N$ [2], [3]. If (N-3) is prime, N also has (P+3) prime pairs (see Table 2).

If N can be divisible by both 2 and 3, but not divisible by any other prime number greater than 3 and less than $\sqrt{N}$, then $D_N \approx L_N$ [1] (see Table 3).

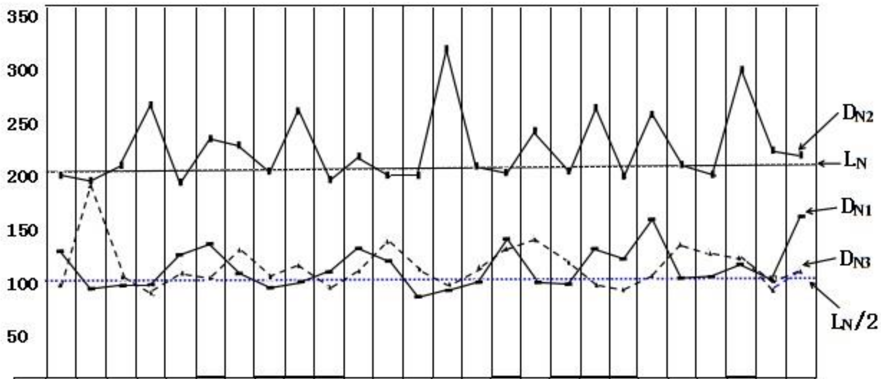
If N is also divisible by other primes greater than 3 and less than $\sqrt{N}$, then the P(1,1) primes of N will be paired with the number, at least increasing the reciprocal times of these prime factors [1], [2], [3], based on being divisible by 2 but not by 3, or being divisible by 2 and also by 3. (see Table 1and Table 4).

Based on classification, regardless of the influence of the prime factor, the larger N is the more P (1,1) prime number pairs there are (see Table 2 and Table 3).

4. The Formulas between the number of P (1, 1) prime pairs and the number of twin prime pairs smaller than the even number.

Suppose that the P(1,1) prime pairs of N_1 has D_{N1} groups (real groups, but not including (P+3) prime pairs, the same below), and the twin prime numbers smaller than N_1 have L_{N1} groups, then $D_{N1} < 0.5L_{N1}$. Suppose that the P (1,1) primes of N_2 have D_{N2} groups, and the twin primes smaller than N_2 have L_{N2} groups, then $D_{N2} < L_{N2}$. Suppose the P(1,1) prime number pairs of N_3 has D_{N3}

groups, and the twin prime number smaller than N_3 has L_{N3} group, then D_{N3} ‹ $0.5L_{N3}$.



m	1667	1668	1669	1670	1671	1672	1673	1674	1675	1676	1677	1678	1679	1680	1681	1682	1683	1684	1685	1686	1687	1688	1689	1690	1691	1692
D_{N1}	127	92	98	99	121	139	109	96	102	109	131	119	91	96	101	136	101	99	124	115	157	100	101	109	100	162
D_{N2}	197	192	209	263	194	238	235	203	256	190	218	198	198	315	208	200	241	201	254	194	244	204	194	292	214	212
D_{N3}	99	190	104	93	106	104	131	105	113	99	111	136	110	98	109	119	131	108	97	96	106	124	113	110	95	106
L_N	204	204	205	205	205	205	205	206	206	206	206	206	207	207	207	207	208	208	208	208	208	208	208	208	209	209
$L_N/2$	102	102	103	103	103	103	103	103	103	103	103	103	104	104	104	104	104	104	104	104	104	104	104	104	105	105

Figure 2: D_{N1} curve, D_{N2} curve, D_{N3} curve (L_N calculated by N=6m)

It is expressed by a unified Formulas: Suppose the P (1,1) prime number of N has D_N group, the twin prime number less than N has L_N group, and R is the remainder of $\frac{N}{6}$, then D_N ‹ $L_N \times \frac{1+|3-R|}{4}$.

If the number of twin primes L_N less than N is known, we can also use L_N to approximate the number of P(1,1) prime pairs of N. Based on the comprehensive analysis of 10 to more than 100 million even numbers, the theoretical calculation value D_S of P(1,1) prime pairs of N is approximated by the following formula:

If N factorial P_1, P_2, P_3..., let $k=(L_N \times 0.988 \times \frac{1+|3-R|}{4})$(Rounding),

$$u=((k \times \frac{1+3 \div P_1}{P_1})\text{ (Rounding)}+(k \times \frac{1+3 \div P_2}{P_2})\text{(Rounding)}+(k \times \frac{1+3 \div P_3}{P_3})\text{(Rounding)}+\ldots\ldots),$$

$$v=((k \times \frac{2}{P_1 \times P_2})\text{(Rounding)}+(k \times \frac{2}{P_1 \times P_3})\text{(Rounding)}+(k \times \frac{2}{P_2 \times P_3})\text{(Rounding)}+\ldots\ldots),$$

$D_S \approx k+u+v$ (see Table 1, 4)

(Note: a. The combined effect of 3 or more prime factors accounts for a small proportion of D_S and is ignored. b. Because it is an approximate calculation, it can also be rounded in the last step, and the difference between the two integer methods does not exceed 2 times the number of prime factors.)

For large or infinite even numbers, although it is not yet certain that the number of twin primes is infinite, the known twin primes will not disappear. The number of P(1,1) prime pairs of infinite even numbers must not be less than half of the number of known twin primes [1], [2], [3].

5. Conclusion

Even numbers greater than or equal to 6 can be expressed as the sum of two prime numbers, and the Goldbach Conjecture holds.

Table 1: The ratio of consecutive even D_N to L_N, and the coincidence rate of the theoretically calculated value D_S to the real value D_N (L_N=440312)

N	D_N	D_N/L_N (%)	Prime factor	k	u	v	D_S	D_S/D_N (%)
100000000	291400	66.2	5	217514	69604		287118	98.5
100000002	464621	105.5	19, 739, 1187	435028	27469	99	462596	99.6
100000004	247582	56.2	13, 29, 5101	217514	28911	1161	247586	100.0
100000006	218966	49.7	491	217514	445		217959	99.5
100000008	437717	99.4		435028			435028	99.4
100000010	323686	73.5	5, 11	217514	94770	7909	320193	98.9
100000012	263241	59.8	7	217514	44390		261904	99.5
100000014	437518	99.4		435028			435028	99.4
100000016	220846	50.2	97	217514	2311		219825	99.5
100000018	233634	53.1	17, 1451, 2027	217514	15309	29	232852	99.7
100000020	595554	135.3	5, 47	435028	149054	3702	587784	98.7
100000022	220244	50.0	103	217514	2173		219687	99.7
100000024	218846	49.7		217514			217514	99.4
100000026	537452	122.1	7, 43	435028	99603	2890	537521	100.0
100000028	220614	50.1	113	217514	1976		219490	99.5
100000030	318202	72.3	5, 13	217514	90197	6692	314403	98.8
100000032	488938	111.0	11, 281, 337	435028	53199	524	488751	100.0
100000034	218651	49.7		217514			217514	99.5
100000036	218867	49.7		217514			217514	99.4

Table 2: Comparison of L_{N1}, D_{N1}, D_S of N_1 even numbers that are divisible only by 2 and cannot be divisible by prime numbers greater than 2 and less than $\sqrt{N_1}$ (wis still existing (P+3) prime pairs, not in the D_{N1} range)

N_1	w	L_{N1}	D_{N1}	D_{N1}/L_{N1} (%)	D_S	D_S/D_{N1} (%)
16	1	2	1	50.00	1	100.00
64	1	6	4	66.67	2	50.00
256	0	16	8	50.00	7	87.50
1024	1	35	21	60.00	17	80.95
4096	1	106	52	49.06	52	100.00
16384	1	289	150	51.90	142	94.67
65536	0	859	435	50.64	424	97.47
262144	0	2678	1314	49.07	1322	100.61
1048576	1	8534	4238	49.66	4215	99.46
4194304	1	27994	13704	48.95	13829	100.91
16777216	1	92245	45745	49.59	45569	99.62
67108864	0	309560	153850	49.70	152922	99.40

Table 3: Comparison of L_{N2}, D_{N2}, and D_S when N_2 is multiplied without prime factor influence

N_2	L_{N2}	D_{N2}	D_{N2}/L_{N2} (%)	D_S	D_S/D_{N2} (%)
24	3	3	100.00	2	66.67
96	7	7	100.00	6	85.71
384	20	19	95.00	19	100.00
1536	49	47	95.92	48	102.13
6144	144	145	100.69	142	97.93
24576	405	397	98.02	400	100.76
98304	1204	1185	98.42	1189	100.34
393216	3750	3679	98.11	3705	100.71
1572864	12088	11952	98.87	11942	99.92
6291456	39529	38887	98.38	39054	100.43
25165824	131253	129941	99.00	129676	99.80
100663296	442913	439602	99.25	437598	99.54

Table 4: The effect of a prime factor on the number of P (1,1) primes of N_2 on the number of prime pairs

N_2	L_{N2}	D_{N2}	D_{N2}/L_{N2} (%)	Prime factor	k	u	v	D_S	D_S/D_{N2} (%)
6126120	38625	78724	203.82	5,7,11,13,17	38161	30665	9396	78222	99.36
6068370	38287	69916	182.61	5,7,11,37,71	37827	25859	5895	69581	99.52
6128640	38636	64631	167.28	5,7,19	38172	22331	3558	64061	99.12
6881280	42610	67264	157.86	5,7	42098	22062	2405	66565	98.96
7249920	44576	59848	134.26	5,59	44041	14877	298	59216	98.94
5635320	35932	47509	132.22	5,151,311	35500	11714	141	47355	99.68
7864320	47902	62982	131.48	5	47327	15144		62471	99.19
6075000	38326	50359	131.39	5(5,5,5,5)	37866	12117		49983	99.25
6537216	40831	51335	125.73	7,19	40341	10690	606	51637	100.59
5083008	32892	40914	124.39	7,31,61	32497	8339	485	41321	100.99
5074944	32847	39427	120.03	7,59	32452	7200	157	39809	100.97
5505024	35204	41612	118.20	7	34781	7098		41879	100.64
5531904	35363	41931	118.57	7(7,7,7)	34938	7130		42068	100.33
6488064	40544	44536	109.85	11	40057	4634		44691	100.35
6324912	39705	43387	109.27	11(11,11,11)	39228	4538		43766	100.87
6094848	38432	39121	101.79	31	37970	1343		39313	100.49
5541126	35417	36197	102.20	31(31,31,31)	34991	1237		36228	100.09

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