Transactions on Computational and Applied Mathematics (2024)
Clausius Scientific Press, Canada

DOI: 10.23977/tracam.2024.040118
ISSN 2616-1826 Vol. 4 Num. 1

An improved kernel regularized nonhomogeneous grey model based on conformable fractional accumulation

Wujian Rao[1,#], Panlin Li[2,#]

[1]*School of Mathematics, Yunnan Normal University, Kunming, 650500, China*
[2]*College of Mathematics and Physics, Xinjiang Agricultural University, Urumqi, 830052, China*
[#]*These authors contributed equally.*

Keywords: Grey Model; Kernel Regularization; Hiking Optimization Algorithm; Energy Comsumption Forecasts

Abstract: In this work, we propose an improved grey prediction model, called the conformable fractional accumulation kernel regularized nonhomogeneous grey model(CFAKRNGM), for energy consumption forecasting, and using HOA to optimize the model parameters, the CFAKRNGM model shows significant advantages in data fitting and prediction accuracy. In order to verify the effectiveness of the model, this work compares the prediction results of the CFAKRNGM model and the KRNGM model through the data analysis of renewable energy consumption and natural gas consumption in China. To provide theoretical support and practical reference for China's energy production and consumption planning. The results explain that the CFAKRNGM model performs better than the KRNGM in dealing with complex and nonlinear time series data, and has higher prediction accuracy and reliability.

1. Introduction

In recent years, research on energy forecasting has evolved. The application of intelligent network model, statistical model and grey model in this field has achieved remarkable results. The grey model has been attracting extensive research by scholars at home and abroad. The grey system theory was proposed by Deng[1], which is a new theory to deal with the problem of uncertainty. The grey prediction model is the middle of the grey system theory, and it is very effective in small-sample time series forecasting, among which the GM(1,1) model has been widely used.

The new grey model obtained by improving the GM(1,1) model has been verified and accepted to real life production. For example, Zhou et al[2]designed a new discrete grey model for predicting natural gas consumption in Jiangsu Province, China. Cheng et al[3]predicted China's clean energy consumption in 2025 by improving the grey model GM(1,N), demonstrating the role of economic growth in driving clean energy demand. However, when the grey system adds more variable information, the solution of the existing grey prediction model GM(1,N) is inaccurate, resulting in unpredictable prediction accuracy. In addition, Ding et al[4]proposed a novel optimized grey prediction model NOGM(1,1) for predicting electricity consumption in China, demonstrating its superiority in dealing with complex nonlinear time series. However, when the multi-parameter form appears in the system and contains nonlinear structures, the effectiveness of the NOGM(1,1) model is greatly reduced. The kernel method in machine learning theory has significant advantages in dealing with nonlinear pattern analysis problems. Duan et al[5]proposed GMC(1,N) model suitable for nonlinearity based on the Gaussian kernel function and the polynomial kernel function, combined with the characteristics of the grey prediction model. In addition, Liu et al[6]used the adjacency nonhomogeneous grey model to predict renewable energy consumption in Europe and the world, and used heuristic algorithms to optimize the parameters, which significantly improved the prediction accuracy.

In the course of the research, the combination of grey prediction models and other models was further introduced to predict energy consumption and production. Wu and Shen[7]proposed a novel LSSVM model[7]combined with grey relationship analysis (GRA) and LSSVM. the new model is abbreviated as GRA-LSSVM, which is used to get more accurate results. The above research on grey model has gradually riched the grey system theory and enhanced the prediction ability to a certain extent. However, in the face of some nonlinear and fluctuating system sequences, it is difficult for the existing grey model and its extension to provide satisfactory accuracy due to its linear structure. Moreover, the existing non-homogeneous grey models are inefficient in predicting complex nonlinear time series. Therefore, the model structure must be improved to further solve the problem of system prediction characterized by nonlinearity and fluctuation.

In this paper, the CFAKRNGM model is established by combining the KRNGM model with the conformable fractional accumulation, and the effectiveness of the proposed model is verified by predicting the change trend of China's renewable resource consumption and China's natural gas consumption. To provide theoretical support and practical reference for energy production and consumption planning, so as to promote the healthy and sustainable development of China's energy.

The remainder of this article is arranged as follows. Section 2 briefly introduces the construction principle and parameter estimation process of the original KRNGM model. Section 3 describes the modeling procedure for the CFAKRNGM model. Section 4 introduces the hyperparameter optimization of the CFAKRNGM model and the HOA algorithm. Section 5 shows the cases studies of predicting the renewable energy consumption and natural gas consumption in China. Conclusions of the study are shown in Section 6.

2. The KRNGM model

2.1. Representation of the KRNGM model

The 1-AGO series of the original series $\left(\mathrm{x}^{(0)}(1), \mathrm{x}^{(0)}(2), \ldots, \mathrm{x}^{(0)}(\mathrm{n})\right)$ is noted as $\left(\mathrm{x}^{(1)}(1), \mathrm{x}^{(1)}(2), \ldots, \mathrm{x}^{(1)}(\mathrm{n})\right)$, where

$$\mathrm{x}^{(1)}(\mathrm{j}) = \sum_{\mathrm{k}=1}^{\mathrm{j}} \mathrm{x}^{(0)}(\mathrm{k}).$$

The differential equation (See[8])

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = f(t) + c \tag{1}$$

is called the KRNGM model, which can also be expressed as(See[8])

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = w^T\varphi(t) + c. \tag{2}$$

2.2. Parameters estimation for the KRNGM model

We need to consider the integration of equation(3) to estimate the parameters of the KRNGM model[8], in the interval $[j-1, j]$, we have[8]

$$\int_{j-1}^{j} d\,x^{(1)}(t) + a\int_{j-1}^{j} x^{(1)}(t)dt = w^T \int_{j-1}^{j} \psi(t)dt + \int_{j-1}^{j} c\,dt. \tag{3}$$

Noticing that $\int_{j-1}^{j} d\,x^{(1)}(t) = x^{(1)}(j) - x^{(1)}(j-1) = x^{(0)}(j)$ and $\int_{j-1}^{j} c\,dt = c$, using the two point trapezoid formula, we have

$$x^{(0)}(j) + az^{(1)}(j) = w^T\Psi(j) + c, \tag{4}$$

$$z^{(1)}(j) = \frac{1}{2}\left(x^{(1)}(j) + x^{(1)}(j-1)\right)$$

$$\Psi(j) = \frac{1}{2}\left(\psi(j) + \psi(j-1)\right).$$

The parameters can be estimated by follows.(See[8])

$$\min J(a, w, c; e) = \frac{a^2}{2} + \frac{w^T w}{2} + \frac{\gamma}{2}\sum_{i=2}^{r} e_i^2, \tag{5}$$

where γ is the regularized parameter.

Define the Lagrangian function as

$$L := \frac{a^2}{2} + \frac{w^T w}{2} + \frac{\gamma}{2}\sum_{i=2}^{n} e_i^2 + \sum_{j=2}^{n} \lambda_j \left[x^{(0)}(k) + az^{(1)}(k) - w^T\Psi(k) - c - e_i\right], \tag{6}$$

then the KKT conditions[8] is given as

$$\begin{pmatrix} 0 & \vec{\mathbf{1}}_{n-1}^{T} \\ \vec{\mathbf{1}}_{n-1} & \Omega + \gamma^{-1} I_{n-1} \end{pmatrix} \begin{pmatrix} c \\ \vec{\lambda} \end{pmatrix} = \begin{pmatrix} 0 \\ Y \end{pmatrix}, \tag{7}$$

where

$$\begin{aligned} \vec{\mathbf{E}}_{n-1} &= [1,1,\ldots,1]_{n-1}^{T}, \\ \Omega &= \left(\psi(i)\cdot\psi(k) - z^{(1)}(i)z^{(1)}(k)\right)_{(n-1)\times(n-1)} \\ \vec{\lambda} &= [\lambda_2, \lambda_3, \ldots, \lambda_n]^T, \\ Y &= \left[x^{(0)}(2) - x^{(0)}(1), x^{(0)}(3) - x^{(0)}(2), \ldots, x^{(0)}(n) - x^{(0)}(n-1)\right]^T, \end{aligned} \tag{8}$$

combine with the definition of ψ, we have

$$\Psi(i)\cdot\Psi(j) = \frac{1}{4}\left(\psi(i) + \psi(i-1)\right)\cdot\left(\psi(j) + \psi(j-1)\right). \tag{9}$$

The kernel function[8] $K(\cdot,\cdot)$ is

$$\psi(i)\cdot\psi(j) = \mathrm{H}(i,j) = \mathrm{H}_{ij}, \tag{10}$$

then the $\psi(i)\cdot\psi(j)$ can be written as

$$\Psi(i)\cdot\Psi(j) = \frac{1}{4}\left(\mathrm{H}_{ij} + \mathrm{H}_{i-1,j} + \mathrm{H}_{i,j-1} + \mathrm{H}_{i-1,j-1}\right). \tag{11}$$

The gaussian kernel is

$$\mathrm{H}(i,j) = \exp\left\{-\frac{||i-j||^2}{2\sigma^2}\right\}, \tag{12}$$

where the σ is the kernel parameter.
The differential equation can be solved as (See[8])

$$\hat{x}^{(1)}(t) = x^{(0)}(1)e^{-a(t-1)} + \int_1^t e^{-a(t-\tau)}\Phi(\tau)d\tau\,, \tag{13}$$

where

$$\Phi(t) = w^T\psi(t) + c, \tag{14}$$

the nonlinear function $w^T\varphi(t)$ can be rewritten as (See[8])

$$w^T\psi(t) = \left[\sum_{j=2}^{n} \lambda_j \Psi(j)\right] \cdot \psi(t) = \frac{1}{2}\sum_{j=2}^{n} \lambda_j[\psi(j) + \psi(j-1)] \cdot \psi(t), \tag{15}$$

we have

$$w^T\psi(t) = \frac{1}{2}\sum_{j=2}^{n} \lambda_j[\psi(j) + \psi(j-1)] \cdot \psi(t) = \frac{1}{2}\sum_{j=2}^{n} \lambda_j\left[\mathrm{H}_{j,t} + \mathrm{H}_{j-1,t}\right], \tag{16}$$

and(See[8])

$$\hat{x}^{(\alpha)}(j) = x^{(\alpha)}(1)e^{-\hat{a}(k-1)} + \frac{1}{2}\sum_{\tau=2}^{k} \left[e^{-\hat{a}(k-\tau)}\Phi(\tau) + e^{-\hat{a}(k-\tau+1)}\Phi(\tau-1)\right], \tag{17}$$

the predicted values can be obtained as

$$\hat{x}^{(\alpha)}(j) = \hat{x}^{(1)}(j) - \hat{x}^{(1)}(j-1). \tag{18}$$

3. The CFAKRNGM model

Obviously, the establishment of the KRNGM model essentially adopts the traditional grey accumulation operator, which still has limitations for small-sample and non-smooth time series prediction. we will improve the original model with a conformable fractional order accumulation operator. The modeling process is very similar to KRNGM, but with slight modifications.

3.1 The conformable fractional accumulation and difference

For all $t > 0, \alpha \in (0,1]$, given a differentiable function $g: [0,\infty) \to R$.Then the CFD of g with α order is defined as

$$T_\alpha(g)(t) = \lim_{\varepsilon\to 0} \frac{g(t+\varepsilon t^{1-\alpha}) - g(t)}{\varepsilon}. \tag{19}$$

Definition 1(See[9])**.** The CFA of g with α order is defined as $\nabla^\alpha g(k) = \nabla\left(\frac{g(k)}{k^{1-\alpha}}\right) = \sum_{i=1}^{k}\left(\frac{g(j)}{j^{1-\alpha}}\right)$, for all $k \in N^+$, $\alpha \in (0,1]$.

Definition 2(See[9])**.** The α order $\big(\alpha \in (n, n+1)\big)$ CFA is defined as

$$\nabla^\alpha g(k) = \nabla^n\left(\frac{g(k)}{k^{[\alpha]-\alpha}}\right). \tag{20}$$

With Definition 2, we can deduce a recursive equality as

$$\nabla^\alpha g(k) = \nabla\left(\nabla^{n-1}\left(\frac{g(k)}{k^{[\alpha]-\alpha}}\right)\right) = \sum_{j=1}^{k}\left(\nabla^{\alpha-1}g(j)\right), \alpha \geq 1, \text{for } n = [\alpha]-1. \tag{21}$$

Combined with the above definitions, we began to build the CFAKRNGM model.

The original non-negative sequence is $X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(N)\right)^T$, N is the length of the sequence, $\alpha \in \mathrm{R}^+$, we have $X^{(\alpha)} = \left(x^{(\alpha)}(1), x^{(\alpha)}(2), \cdots, x^{(\alpha)}(N)\right)^T$, which is the α order accumulation of the conformable fraction of $X^{(0)}$ to generate the sequence, the differential equation

$$\frac{dx^{(\alpha)}(t)}{dt} + \hat{b}x^{(\alpha)}(t) = g(t) + m, \tag{22}$$

is called a congruent fractional-order kernel regularized nonhomogeneous grey model, or CFAKRNGM model for short, the differential equation can also be expressed as

$$\frac{dx^{(\alpha)}(t)}{dt} + \hat{b}x^{(\alpha)}(t) = \omega^T\psi(t) + m. \tag{23}$$

3.2 Parameters estimation for the CFAKRNGM model

Similar to the KRNGM model, equation. (23) needs to be discretized, given that the sample and α, we have

$$\int_{j-1}^{j} dx^{(\alpha)}(t) + \hat{b}\int_{j-1}^{j} x^{(\alpha)}(t)dt = \omega^T\int_{j-1}^{j} \psi(t)dt + \int_{j-1}^{j} mdt, \tag{24}$$

beause $\int_{j-1}^{j} dx^{(\alpha)}(t) = x^{(\alpha)}(j) - x^{(\alpha)}(j-1)$, we have

$$x^{(\alpha)}(j) - x^{(\alpha)}(j-1) + \hat{b}z^{(\alpha)}(j) = \omega^T\Psi(j) + m, \tag{25}$$

where

$$\begin{aligned} z^{(\alpha)}(j) &= \tfrac{1}{2}(x^{(\alpha)}(j) + x^{(\alpha)}(j-1)) \\ \Psi(j) &= \tfrac{1}{2}(\psi(j) + \psi(j-1)) \end{aligned}.$$

Using the following regularization problem to estimate the parameters $\hat{b}, \omega, m$

$$min\Gamma\left(\hat{b}, \omega, m; e\right) = \frac{\hat{b}^2}{2} + \frac{\omega^T\omega}{2} + \frac{\gamma}{2}\sum_{k=2}^{r} e_k^2, \tag{26}$$

where

$$e_k = x^{(\alpha)}(k) + \hat{b}z^{(\alpha)}(k) - \omega^T\Psi(k) - m,$$

the lagrangian function is

$$F := \frac{\hat{b}^2}{2} + \frac{\omega^T\omega}{2} + \frac{\gamma}{2}\sum_{j=2}^{n} e_j^2 + \sum_{j=2}^{n} \lambda_j\left[x^{(\alpha)}(k) + \hat{b}z^{(\alpha)}(k) - \omega^T\Psi(k) - m - e_k\right], \tag{27}$$

The above KKT condition can be translated into.

$$\begin{pmatrix} 0 & \vec{\mathbf{1}}_{n-1}^T \\ \vec{\mathbf{1}}_{n-1} & \xi + \gamma^{-1}I_{n-1} \end{pmatrix}\begin{pmatrix} c \\ \vec{\lambda} \end{pmatrix} = \begin{pmatrix} 0 \\ Y \end{pmatrix}, \tag{28}$$

where

$$\begin{aligned} \vec{\mathbf{E}}_{n-1} &= [1,1,\ldots,1]_{n-1}^T, \\ \xi &= \left(\psi(i)\cdot\psi(k) - z^{(\alpha)}(i)z^{(\alpha)}(k)\right)_{(n-1)\times(n-1)}, \\ \vec{\lambda} &= [\lambda_2, \lambda_3, \ldots, \lambda_n]^T, \\ \boldsymbol{Y} &= \left[x^{(\alpha)}(2) - x^{(\alpha)}(1), x^{(\alpha)}(3) - x^{(\alpha)}(2), \ldots, x^{(\alpha)}(n) - x^{(\alpha)}(n-1)\right]^T. \end{aligned} \tag{29}$$

The condition $x^{(\alpha)}(1) = \nabla\frac{x^{(0)}(1)}{1^{[\alpha]-\alpha}} = x^{(0)}(1)$ can be used to solve Eq.(23), we have

$$\hat{x}^{(\alpha)}(j) = x^{(\alpha)}(1)e^{-\hat{b}(j-1)} + \frac{1}{2}\sum_{\tau=2}^{j}\left[e^{-\hat{b}(j-\tau)}\Phi(\tau) + e^{-\hat{b}(j-\tau+1)}\Phi(\tau-1)\right]. \quad (30)$$

Finally we have

$$\hat{x}^{(0)}(j) = \begin{cases} \hat{x}^{(0)}(j) = j^{1-\alpha}(\hat{x}^{(\alpha)}(j) - \hat{x}^{(\alpha)}(j-1)) & 0 < \alpha \le 1, \\ j^{[\alpha]-\alpha}\Delta^{n}\hat{x}^{(\alpha)}(j) & n < \alpha \le n+1. \end{cases} \quad (31)$$

4. Hyperparameter optimization of the CFAKRNGM models

4.1. HOA algorithm

Oladejo S O et al proposed a new algorithm called "HOA",which is based on the (THF)[10]. The process as follows:

$$\Phi_{i,t} = 6e^{-3.5|S_{i,t}+0.05|}, \quad (32)$$

$W_{i,t}$ is the i velocity(km/h) for the subject of study.(See[10]).

$$S_{i,t} = \frac{\mathrm{dy}}{\mathrm{dx}} = \tan\varpi_{i,t}\,, \quad (33)$$

where dy and dx are the difference in elevation and distance. In addition, $\varpi_{i,t}$ is the inclination of the trail or terrain. (See[10]).

$$\mathcal{W}_{i,t} = \mathcal{W}_{i,t-1} + \mu_{i,t}(\beta_{best} - \alpha_{i,t}\beta_{i,t}), \quad (34)$$

where $\mu_{i,t}$ is a uniformly distributed number, $\mathcal{W}_{i,t}$ and $\mathcal{W}_{i,t-1}$ represent the current and initial speed , β_{best} is the position of the lead hiker, $\eta_{i,t}$ is the swept factor of ascender i.

$$\beta_{i,t+1} = \beta_{i,t} + \Phi_{i,t}, \quad (35)$$

$\beta_{i,t+1}$ Indicates the updated location of Hiker i. The initialization of the hiker position is represented by the equation

$$\beta_{i,t} = \chi_j^1 + \delta_j(\chi_j^2 - \chi_j^1), \quad (36)$$

where δ_j is the number of uniform distributions. χ_j^1 and χ_j^2 denote the lower and upper bound of the j-dimension(See[10]).

4.2. Optimization strategy for model hyperparameters

ALGORITHM 1: Algorithm of HOA to search for the parameters of the CFAKRNGM model	
	Input: The raw data $X^{(0)}, ub, lb, objfun, nhikers, ndim, MaxIter, nn$
	Output: $\boldsymbol{\sigma}, \boldsymbol{\gamma}, \boldsymbol{\alpha}$
1	Initialize the MaxIter and the number of hikers
2	Initialize the hikers' position randomly.
3	**for** $j = 1;\ j < T;\ j = j + 1$ **do**
4	**for** $each\ hiker$ **do**
5	Update the parameters $\boldsymbol{\sigma}', \boldsymbol{\gamma}', \boldsymbol{\alpha}'$
6	Compute $\hat{x}^{(\alpha)}(j)$ using Eq.(30)
7	Compute $\hat{x}^{(0)}(j)$ using Eq.(31);
8	**end**

Compute MAPE using Eq.(37)
if $MAPE < MAPE_{min}$ **then**
$MAPE_{min} \leftarrow MAPE$
$\boldsymbol{\alpha}' \leftarrow \alpha, \boldsymbol{\gamma}' \leftarrow \boldsymbol{\gamma}, \boldsymbol{\sigma}' \leftarrow \boldsymbol{\sigma}$
end
end
return $\boldsymbol{\sigma}', \boldsymbol{\gamma}', \boldsymbol{\alpha}'$

Then we need to build a simple optimization question to determine the optimal value of the parameters as follows.

$$\min_{\alpha} MAPE = \sum_{j=1}^{N} \left| \frac{\hat{x}^{(\alpha)}(j) - x^{(\alpha)}(j)}{x^{(\alpha)}(j)} \right| \times 100\% \tag{37}$$

This formula is typically used for nonlinear grey models with tunable parameters to minimize the MAPE as an objective function.

5. Numerical example

This section evaluates the effectiveness of the CFAKRNGM model in terms of renewable and non-renewable resources through case studies of renewable energy consumption and natural gas consumption in China. The results of the CFAKRNGM model were compared with the KRNGM to test the prediction performance of the CFAKRNGM model.

The minimize MAPE and the average relative error(ARE) are used to test the accuracy of the model, the ARE is defined as follows

$$\text{ARE} = \frac{1}{N} \sum_{j=1}^{N} \left| \frac{\hat{x}^{(\alpha)}(j) - x^{(\alpha)}(j)}{x^{(\alpha)}(j)} \right| \times 100(\%).$$

Case 1: Short-term consumption forecast for renewable energy in China

In this case, the CFAKRNGM model and the KRNGM model are used to predict the consumption of renewable resource in China, respectively. The data comes from the energy institute (https://www.energyinst.org/statistical-review), combined with the timeliness of energy data and the characteristics of the exponential distribution of small sample series in the gray model, we collect 20 sample datas from 2004 to 2023 and reports them in Table 1. In order to visually express the validity and authenticity of the model, the prediction error is reduced. A total of 16 sets of data from 2004 to 2019 are used for the fitting phase to build a prediction model. Data from 2020 and 2023 are used to verify model accuracy.

Table 1: China's renewable resource consumption from 2004 to 2023.

Year	Consumption of renewable resource	Year	Consumption of renewable resource
2004	3.73	2014	12.79
2005	4.20	2015	13.71
2006	4.36	2016	14.89
2007	5.16	2017	16.22
2008	6.81	2018	17.79
2009	6.78	2019	19.47
2010	7.97	2020	21.03
2011	7.99	2021	23.51
2012	10.01	2022	25.55
2013	10.92	2023	27.60

Table 2: In case 1, the optimal tunable parameters of the two models.

Model	$\boldsymbol{\sigma}$	$\boldsymbol{\gamma}$	$\boldsymbol{\alpha}$
CFAKRNGM	1.0706	206.9354	0.9998
KRNGM	0.9563	3.9364	

By calculating and comparing the MAPE of the two models, the optimal tunable parameters found by the hiking optimization algorithm are shown in Table 2.

Table 3: Model prediction results and errors in Case 1

Year	**Raw data**	**CFAKRNGM**	**KRNGM**
2004	3.73	3.7300	3.7300
2005	4.20	4.1777	4.2352
2006	4.36	4.5566	4.6298
2007	5.16	5.3171	5.4113
2008	6.81	6.5278	6.3625
2009	6.78	7.0989	7.0859
2010	7.97	7.6373	7.6546
2011	7.99	8.3389	8.4618
2012	10.01	9.6868	9.6918
2013	10.92	11.1758	11.1514
2014	12.79	12.5992	12.5551
2015	13.71	13.8493	13.7713
2016	14.89	14.8429	14.9248
2017	16.22	16.2537	16.2356
2018	17.79	17.7720	17.7594
2019	19.47	19.4710	19.5722
ARE(fit)		0.0012	0.0013
MAPE(fit)		0.0195	0.0220
2020	21.03	21.5930	21.7910
2021	23.51	24.1850	24.3443
2022	25.55	27.0879	27.1067
2023	27.60	30.1686	30.0939
ARE(pre)		0.0130	0.0139
MAPE(pre)		0.0522	0.0557

The prediction results and errors of the fitting and testing phases are shown in Table 3. The Table 2 shows that the three optimal tunable parameters obtained in the CFAKRNGM model are α=0.9998, δ=1.0706 and γ=206.9354. Meanwhile, we can obtain from the evaluation index values of the two models in Table 3 that the fitting and prediction accuracy of the two types of grey prediction models are relatively high, but the MAPE and the ARE of the CFAKRNGM model is smaller than that of KRNGM in the fitting and testing stages of the model. This indicates that the prediction accuracy and fitting accuracy of the CFAKRNGM model are higher than those of the KRNGM model.

Figure 1 shows the data fitting plot lines for both models. From this, we can see that the data change trend of the two curves is similar to the exponential curve. Combined with Table 3, we can clearly get that the ARE and MAPE of the CFAKRNGM model are smaller than those of the KRNGM model in both the fitting and prediction stages, and the predicted datas of the CFAKRNGM model are closer to the true datas, the prediction error is also smaller.

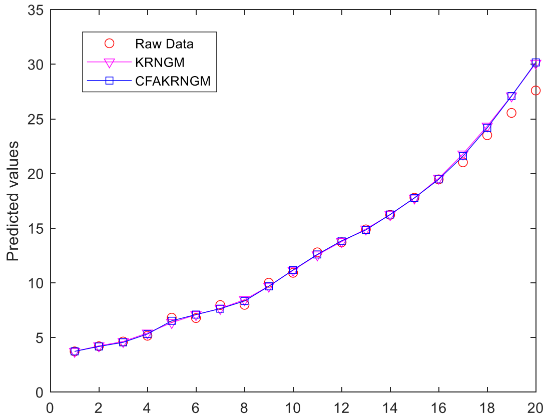

Figure 1: In Case 1, Fitting curves for CFAKRNGM and KRNGM model data

Case 2: Short-term natural gas consumption forecast in China

Case 1 fits and predicts data that conform to the approximate exponential distribution pattern. Case 2 analyzes and forecasts the natural gas consumption from 2004 to 2023 in China, with the original data shown in Table 4. There are 20 sets of data, the first 16 sets of data are used to build a prediction model, and the remaining 4 sets of data are used to verify the prediction accuracy of the model.

Table 4: China's natural gas consumption from 2004 to 2023

Year	**Natural gas consumption**	**Year**	**Natural gas consumption**
2004	1.44	2014	6.78
2005	1.69	2015	7.01
2006	2.08	2016	7.54
2007	2.56	2017	8.69
2008	2.95	2018	10.22
2009	3.25	2019	11.10
2010	3.92	2020	12.12
2011	4.87	2021	13.69
2012	5.43	2022	13.60
2013	6.19	2023	14.57

The optimal tunable parameters obtained by the HOA are list in Table 5, and the prediction results and errors of the fitting and testing stages of two models are shown in Table 6. From the table below, we can get that the three optimal tunable parameters obtained in the CFAKRNGM model are α=0.9998, δ=1.0706 and γ=206.9354.

Table 5: The optimal tunable parameters of the two models in Case 2

Model	σ	γ	α
CFAKRNGM	1.1318	12.9982	0.9995
KRNGM	1.0054	3.9156	

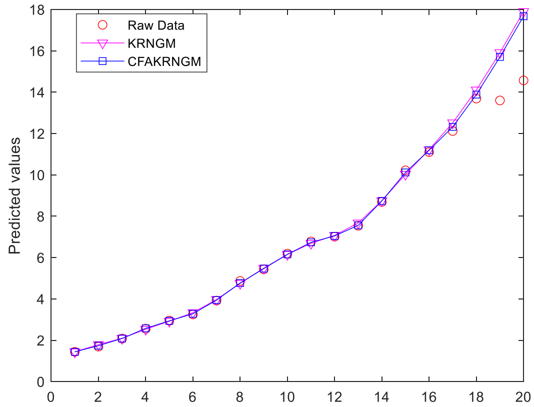

Figure 2: The CFAKRNGM and KRNGM models fit the curves in Case 2

The fitting and prediction accuracy of the CFAKRNGM model have been improved by different amplitudes, the MAPE and ARE is lower. The prediction effect is obviously better.

Table 6: Model prediction results and errors in Case 2

Year	**Raw data**	**CFAKRNGM**	**KRNGM**
2004	1.44	1.4400	1.4400
2005	1.69	1.7373	1.7905
2006	2.08	2.0864	2.0946
2007	2.56	2.5776	2.5320
2008	2.95	2.9381	2.9206
2009	3.25	3.2806	3.3351
2010	3.92	3.9427	3.9714
2011	4.87	4.7698	4.7506
2012	5.43	5.4675	5.4862
2013	6.19	6.1531	6.1552
2014	6.78	6.7380	6.6870
2015	7.01	7.0461	7.0735
2016	7.54	7.5606	7.6723
2017	8.69	8.7395	8.7508
2018	10.22	10.1184	10.0271
2019	11.10	11.1905	11.2202
ARE(fit)		0.0005	0.0009
MAPE(fit)		0.0080	0.0153
2020	12.12	12.3225	12.5183
2021	13.69	13.8816	14.1051
2022	13.60	15.7065	15.9124
2023	14.57	17.6733	17.9008
ARE(pre)		0.0025	0.0029
MAPE(prediction)		0.0996	0.1155

The data given in Case 2 are the same as those in Case 1, and they are distributed according to the approximate exponential law. Figure 2 shows the data fitting plot lines for both models. From this, we can intuitively see that the two curves move in about the same direction, but the data curve of the CFAKRNGM model is obviously closer to the true value. Combined with the results of the model in Table 6 and data curves given in Figure 2, it can be clearly seen that the prediction results of the CFAKRNGM model are closer to the true value than the KRNGM.

6. Conclusion

For data series that are easily affected by complex factors, the CFAKRNGM model effectively overcomes the bias of the data in the prediction and fitting stages of the KRNGM model. Two practical cases verify the theoretical results of this paper and the rationality of the model. The results explain that the CFAKRNGM model can better reflect the data development trend, and the fitting and prediction accuracy are improved compared with the KRNGM model. Compared with the existing models, the proposed CFAKRNGM model is more effective in long-term forecasting and forecasting for unstable time series. In real world, the development of a system not only has to consider the influence of its own historical factors, but also other uncontrollable factors, and may show the characteristics of instability, periodicity, nonlinearity, etc., which makes it very difficult to accurately predict the system. Therefore, in future research, the Hiking Optimization Algorithm (HOA) can be considered to be combined with the multivariate hyperparameter grey prediction model to solve the influence of uncontrollable factors on multivariate data, and improve the accuracy and practicability of the model.

Acknowledgments

This work was financially supported by 2024 Xinjiang Agricultural University Student Entrepreneurship Programs (S202410758002X); 2024 Xinjiang Agricultural University Student Innovation Programs(dxscx2024600) fund.

References

[1] Deng Julong. Grey System Theory Tutorial[M].Wuhan Huazhong University of Science and Technology Press,1992
[2] Zhou W, Wu X, Ding S, et al. Application of a novel discrete grey model for forecasting natural gas consumption: A case study of Jiangsu Province in China[J]. Energy, 2020, 200: 117443.
[3] Cheng M, Li J, Liu Y, et al. Forecasting clean energy consumption in China by 2025: using improved grey model GM (1, N)[J]. Sustainability, 2020, 12(2): 698.
[4] Ding S, Hipel K W, Dang Y. Forecasting China's electricity consumption using a new grey prediction model[J]. Energy, 2018, 149: 314-328.
[5] Duan H, Wang D, Pang X, et al. A novel forecasting approach based on multi-kernel nonlinear multivariable grey model: A case report[J]. Journal of Cleaner Production, 2020, 260: 120929.
[6] Liu L, Wu L. Forecasting the renewable energy consumption of the European countries by an adjacent non-homogeneous grey model[J]. Applied Mathematical Modelling, 2021, 89: 1932-1948.
[7] Wu Y H, Shen H. Grey-related least squares support vector machine optimization model and its application in predicting natural gas consumption demand[J]. Journal of Computational and Applied Mathematics, 2018, 338: 212-220.
[8] Ma, Xin, Yi-sheng Hu, and Zhi-bin Liu. "A novel kernel regularized nonhomogeneous grey model and its applications." Communications in Nonlinear Science and Numerical Simulation 48 (2017): 51-62.
[9] Ma X, Wu W, Zeng B, et al. The conformable fractional grey system model[J]. ISA transactions, 2020, 96: 255-271.
[10] Oladejo S O, Ekwe S O, Mirjalili S. The Hiking Optimization Algorithm: A novel human-based metaheuristic approach[J]. Knowledge-Based Systems, 2024, 296: 111880.