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Optimization Study of Depth Bomb Hit Probability Based on Monte Carlo Method

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Abstract: This paper focuses on the optimization problem of depth bomb hit probability in submarine anti-submarine warfare [1]. Using the probability model and Monte Carlo method, the relationship between the bomb hit probability and the coordinates of the bomb drop point, and the detonation depth of the fixed depth fuse is analyzed [2,3]. Firstly, a probabilistic model is constructed to generate random samples, and simulation experiments are conducted to determine the maximum value of the probability; secondly, the bombing scheme that maximizes the hit rate of the projectile is found, and the corresponding expression for the maximum hit probability is given; finally, the results show the optimized bombing scheme and its corresponding maximum hit probability under different conditions. It is found that under the condition of error-free positioning, the optimal solution is the specific coordinates and detonation depth, and the hit probability is 0.0346, and under the condition of error-based positioning, the hit probability is 0.0269 after the optimal detonation depth is adjusted, and the hit probability can be increased to 0.2146 by adjusting the detonation depth and the layout of the depth bombs when multiple depth bombs are simultaneously dropped, which is of practical value for enhancing the effectiveness of anti-submarine warfare.

1. Introduction

In submarine anti-submarine warfare, the optimization of the hit probability of deepwater bombs is an important research topic to enhance the effectiveness of the strike. In this paper, an optimization strategy based on probabilistic model and Monte Carlo method is proposed to address this problem, and the optimization model of hit probability is constructed by analyzing the factors such as coordinates of the drop point and plane positioning error. The optimal bomb-dropping schemes under error-free and error-positioning conditions are studied, and the hit probabilities in different cases are calculated. Subsequently, the dropping strategy of multiple deep bombs is adopted to analyze the effects of different array layouts and detonation depths on the hit rate, and the probability of hitting the submarine is successfully maximized. By establishing a mathematical model and using Monte Carlo method to generate random samples, the study finds the detonation depth and drop point with the highest probability of hitting the submarine, and further determines the optimal dropping scheme of the depth bomb. This study provides an effective dropping strategy for anti-submarine warfare in theory, which is of great practical significance for improving the striking effectiveness of anti-submarine warfare.

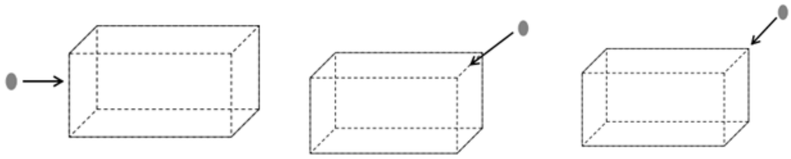
2. Analysis of projectile hit probability under error-free conditions

2.1 Modeling

The purpose of this paper is to establish a mathematical model to calculate the probability of hitting a submarine with a deep bomb and to determine the bombing scheme with the maximum probability of hitting [4]. First of all, define the notation: set the projection point of the submarine's center position in the sea level as the coordinate origin $O(0,0,0)$, X, Y are the horizontal coordinates of the drop point, and Z is the known depth of the submarine, which takes the value of 150 m. The horizontal positioning error δ of the submarine's center position is 120 m, and the size of the submarine is a=100 m (length). The horizontal positioning error δ of the submarine's center position is 120 m. The dimensions of the submarine are a=100 m (length), b=20 m (width), c=25 m (height), and the azimuth angle β=90∘. The kill radius of the depth bomb is r=20 meters, the detonation depth is recorded as d. The real position of the submarine in the horizontal plane obeys the two-dimensional normal distribution $N\ (0, \Sigma)$, and the probability of hitting is recorded as P. The submarine position error model assumes that the actual horizontal position of the submarine obeys the two-dimensional normal distribution $N\ (0, \Sigma)$. The hit probability analysis includes the following three cases: first, the depth of the depth bomb falls within the target plane and detonation depth in the submarine surface below; second, the fall within the target plane and detonation depth in the submarine surface above, the submarine in the kill range; third, the fall outside the target plane and the detonation depth of the submarine in the kill range. The probability of hitting P is expressed as the sum of the probabilities of these three scenarios.

The optimization objective is to maximize the hit rate by determining the optimal drop point (x, y) and the optimal detonation depth d of the fixed depth fuse. First, the optimal drop point (x, y) is chosen to maximize the likelihood that the drop point is within the submarine probability region. Second, the optimal detonation depth, d, is determined to maximize the probability that the submarine will be within the probability area of a depth charge given the drop point (x, y). The maximum hit probability expression is derived by computational optimization and combined with specific parameter values to determine the scenario with the largest bomb drop hit rate, and thus the corresponding maximum hit rate.

This paper systematically analyzes the effect of the explosion of the depth bomb in different directions around the submarine on the hit probability of the submarine. First, when the depth bomb falls within the horizontal range of the submarine and touches the upper surface of the submarine without triggering the depth fuse, the range of values of x and y are determined, at which time the depth bomb explodes directly above the submarine, and the positional relationship between the submarine's center point and the point of explosion as well as its probability of hit are defined. Subsequently, the submarine's due-east direction, due-west direction, due-south direction, due-north direction, as well as the eastward upward, westward downward, eastward downward, westward upward, eastward northward, westward southward, eastward southward, westward northward, northward upward, southward downward, northward downward, southward upward, eastward northeasterly upward, westward southwesterly upward, eastward northeasterly downward, westward southwesterly upward, eastward southeasterly upward, westward northeasterly downward, eastward southeasterly downward and westward northwesterly upward, etc., were considered respectively Twenty-five directions, this paper determines in detail the positional relationship between the submarine center point and the explosion point of the deep bomb in each case, and calculates the corresponding probability of hitting, of which the position of the west, east up and east northeast up direction of the process of solving as Figure 1:



(a) Due west (b) Upper east direction (c) Upper east-northeast direction

Figure 1: Schematic illustration of positions in the west, east-superior and east-northeast-superior directions

Due west:

$$\begin{cases} 0 \le x_1 - \frac{a}{2} - x \le r \\ y_1 - \frac{b}{2} \le y \le y_1 + \frac{b}{2} \\ z_1 - \frac{c}{2} \le z \le z_1 + \frac{c}{2} \end{cases} \tag{1}$$

$$D_{n_3} = \left\{ \mathrm{x} + \frac{a}{2} \le x_1 \le r + x + \frac{a}{2},\ \ \mathrm{y} - \frac{b}{2} \le y_1 \le y + \frac{b}{2} \right\} \tag{2}$$

$$\mathrm{P}_{\mathrm{n}_3} = \iint_{\mathrm{D}_{\mathrm{n}_3}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x_1^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y_1^2}{2\sigma^2}} dx_1 dy_1 \tag{3}$$

Upper east direction:

$$\begin{cases} x_1 + \frac{a}{2} \le x \\ z_1 - \frac{c}{2} \ge \mathrm{z} \\ y_1 - \frac{b}{2} \le y \le y_1 + \frac{b}{2} \\ \left[x - \left(x_1 + \frac{a}{2} \right) \right]^2 + \left(z_1 - \frac{c}{2} - z \right)^2 \le r^2 \end{cases} \tag{4}$$

$$D_{n_6} = \left\{ x_1 \le x - \frac{a}{2},\ \ \mathrm{y} - \frac{b}{2} \le y_1 \le y + \frac{b}{2} \right\} \tag{5}$$

$$\mathrm{P}_{\mathrm{n}_6} = \iint_{\mathrm{D}_{\mathrm{n}_3}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x_1^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y_1^2}{2\sigma^2}} dx_1 dy_1 \tag{6}$$

Upper east-northeast direction:

$$\begin{cases} x_1 + \frac{a}{2} \le x \\ y_1 - \frac{b}{2} \ge y \\ z_1 - \frac{c}{2} \ge z \\ \left(x - x_1 - \frac{a}{2}\right)^2 + \left(y_1 - \frac{b}{2} - y\right)^2 + \left(z_1 - \frac{c}{2} - z\right)^2 \le r^2 \end{cases} \tag{7}$$

$$\mathrm{D}_{n_{18}} = \left\{ x_1 \le x - \frac{a}{2},\ \ \mathrm{y}_1 \ge y + \frac{b}{2} \right\} \tag{8}$$

$$\mathrm{P}_{\mathrm{n}_{18}} = \iint_{\mathrm{D}_{\mathrm{n}18}} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x_1^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{y_1^2}{2\sigma^2}} dx_1 dy_1 \tag{9}$$

Through the comprehensive analysis of the hit probability in each direction, this paper finally proposes the expression of the maximum hit probability of the projectile as follows:

$$\mathrm{P}_{\mathrm{n}_i} = \iint_{\mathrm{D}_{\mathrm{n}i}} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x_1^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{y_1^2}{2\sigma^2}} dx_1 dy_1, i = 1, 2, \dots, 25 \tag{10}$$

$$\mathrm{P}_{\max} = \sum_{i=1}^{25} P_{n_i} \tag{11}$$

2.2 Solving the model

By constructing a probabilistic model to generate random samples to simulate the experiment, Monte Carlo method is applied to analyze the results to find the probability maximum. Figure 1 shows the submarine position and the optimal drop point.

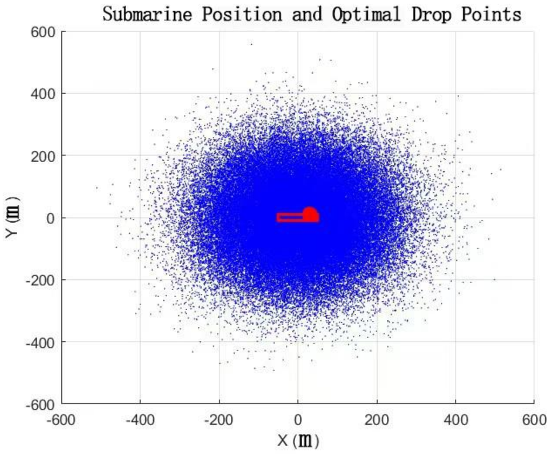


Figure 2: Submarine positions and optimal drop points.

The result is shown in Figure 2: the best drop point is x=10m, y=-5m, d=137.5m, and the hit rate here is 0.0346. Although the hit rate is low, how to improve the hit rate is the direction of this paper's subsequent further research.

3. Calculation of hit probability under the influence of localization error

3.1 Modeling

For the analysis of the probability of hitting a single deep bomb. It is known that the submarine's center position has errors in three directions (X, Y, Z), where the horizontal position error obeys a normal distribution, and the Z-direction position error obeys a one-sided truncated normal distribution. The goal is to select the optimal drop point location and detonation depth, design the optimal expression for the hit probability, and thus determine the explosion coordinates of the depth bomb, to maximize the probability of hitting the submarine.

To build the model, the following are the main analytical steps:

Analyzing the characteristics of the actual depth position: the submarine's depth position error obeys a one-sided truncated-tailed normal distribution, which requires a comprehensive consideration of the depth uncertainty and an accurate calculation of the submarine's probability distribution at different depth positions.

Analyze the characteristics of the actual horizontal position: the horizontal position error obeys the normal distribution with mean value 0 and standard deviation 120, and this error characteristic directly affects whether the depth charge can accurately fall within the horizontal range of the submarine, and its impact on the hitting effect needs to be evaluated in detail.

Determine the best fixed depth detonation depth: combined with the depth of the submarine, the depth of the bomb kill radius and positional error range, optimize the choice of detonation depth, to ensure that the probability of hitting the explosion to maximize the enhancement of combat effectiveness.

Determinants of the probability of hitting a submarine with a deep bomb: The probability of hitting a submarine is affected by the drop point, the choice of plane coordinates and the depth of detonation, and the impact of the change in the proportion of submarines occupying the kill radius on the probability of hitting a submarine needs to be analyzed in depth.

Optimization of detonation depth: By constructing a probability model and generating random samples, Monte Carlo methods are used to conduct an in-depth analysis to determine the detonation depth with the highest probability of hitting, ensuring the optimal design of the combat scheme.

An expression for the probability of hitting:

$$P = \iiint_D \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x_1^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y_1^2}{2\sigma^2}} \frac{1}{\sigma_z} \frac{\varphi\left(\frac{v-h_0}{\sigma_z}\right)}{1-\varphi\left(\frac{l-h_0}{\sigma_z}\right)} dx_1 dy_1 dz_1 \tag{12}$$

There is also a 25 positional relationship between the center of the submarine and the point of explosion of the depth charge, due west and up east, for example:

Due west:

$$\begin{cases} 0 \le x_1 - \frac{a}{2} - x \le r \\ y_1 - \frac{b}{2} \le y \le y_1 + \frac{b}{2} \\ z_1 - \frac{c}{2} \le z \le z_1 + \frac{c}{2} \end{cases} \tag{13}$$

$$D_{m_3} = \left\{ \mathrm{x} + \frac{a}{2} \le x_1 \le r + x + \frac{a}{2},\ \ \mathrm{y} - \frac{b}{2} \le y_1 \le y + \frac{b}{2}, z - \frac{c}{2} \le z_1 \le z + \frac{c}{2} \right\} \tag{14}$$

$$\mathrm{P}_{m3} = \iiint_{D_{m3}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x_1^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y_1^2}{2\sigma^2}} \frac{1}{\sigma_z} \frac{\varphi\left(\frac{v-h_0}{\sigma_z}\right)}{1-\varphi\left(\frac{l-h_0}{\sigma_z}\right)} dx_1 dy_1 dz_1 \tag{15}$$

Upper East Side:

$$\begin{cases} x_1 + \frac{a}{2} \le x \\ z_1 - \frac{c}{2} \ge \mathrm{z} \\ y_1 - \frac{b}{2} \le y \le y_1 + \frac{b}{2} \\ \left[x - \left(x_1 + \frac{a}{2} \right) \right]^2 + \left(z_1 - \frac{c}{2} - z \right)^2 \le r^2 \end{cases} \tag{16}$$

$$D_{m_6} = \left\{ x_1 \le x - \frac{a}{2},\ \ \mathrm{y} - \frac{b}{2} \le y_1 \le y + \frac{b}{2}, z_1 \ge z + \frac{c}{2} \right\} \tag{17}$$

$$\mathrm{P}_{m_6} = \iiint_{D_{m_6}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x_1^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y_1^2}{2\sigma^2}} \frac{1}{\sigma_z} \frac{\varphi\left(\frac{v-h_0}{\sigma_z}\right)}{1-\varphi\left(\frac{l-h_0}{\sigma_z}\right)} dx_1 dy_1 dz_1 \tag{18}$$

The expression for the maximum hit probability of a projectile is:

$$\mathrm{P}_{m_i} = \iiint_{D_{mi}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x_1^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y_1^2}{2\sigma^2}} \frac{1}{\sigma_z} \frac{\varphi\left(\frac{v-h_0}{\sigma_z}\right)}{1-\varphi\left(\frac{l-h_0}{\sigma_z}\right)} dx_1 dy_1 dz_1, i = 1, 2, \ldots, 25 \tag{19}$$

$$\mathrm{P}_{\max} = \sum_{i=1}^{25} P_{m_i} \tag{20}$$

3.2 Solving the model

By constructing a probabilistic model to generate random samples to simulate the experiment, Monte Carlo method is applied to analyze the results to find the probability maximum. Figure 3 shows the submarine position and the optimal drop point [5].

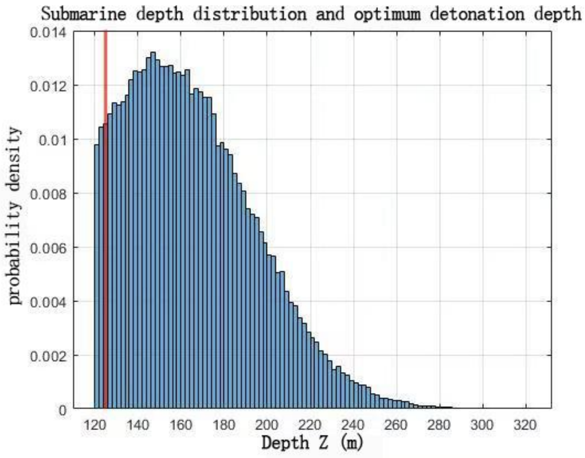


Figure 3: Submarine depth distribution and optimum depth of detonation.

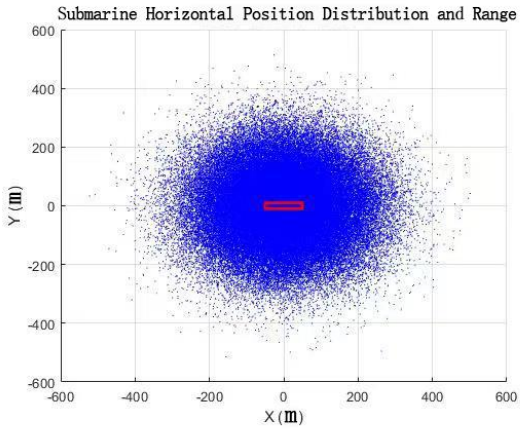


Figure 4: Distribution of horizontal positions of submarines

Figure 3 illustrates the probability distribution of submarine depths and the optimal detonation depth. The depth distribution is characterized by a one-sided truncated normal distribution, concentrated between 120 and 300 meters, with the probability density peaking at 140 to 180 meters. This indicates that the submarine is more likely to be present at these depths. The optimal detonation depth is marked by a red vertical line, which is in the high probability region, indicating that detonation at this depth can significantly increase the probability of hit and optimize operational effectiveness.

Figure 4 shows the distribution of the submarine's horizontal position, which is presented as a centrosymmetric normal distribution ranging from -600 m to 600 m. The dense blue scatter shows the wide distribution of the submarine in the horizontal position. The red rectangular box marks the higher probability area where the submarine may appear, indicating that deep bombing in this range can effectively cover the horizontal position of the submarine. Combining the two figures, the analysis shows that precise depth selection and horizontal position coverage are crucial for improving the hit probability, emphasizing the necessity of precise positioning and depth control in submarine operations. In summary, the results show that the optimal depth of detonation is 125 meters, and the maximum hit probability is 0.0269.

On this basis, the study focuses on analyzing the dropping strategy of multiple depth charges for the problem of maximizing the hit probability of submarines. All the depths of detonation of the depth bombs are the same, and the drop positions are in the shape of an array. The key is to select the optimal drop location and detonation depth to design the best solution to ensure that at least one depth charge hits the submarine. The core elements of the solution included the array layout and the choice of depth of detonation. Nine depth charges are to be dropped in a square, rectangular or hexagonal array, with the shape and spacing of the array directly affecting the probability of a hit. The depth of detonation should be in a high probability region of the submarine's possible depths, based on previous analysis, to maximize the probability of hit. The probability of at least one hit is determined by calculating the probability of a miss for each depth charge.

The modeling steps include selecting a suitable array layout, determining the center position, setting reasonable intervals to cover a large area and increase the hit probability; selecting the optimal detonation depth, located in an area with high submarine distribution probability; and calculating the probability of at least one depth charge hit, using the combined probability formula, solved by opposites. The mathematical model is defined as follows: the number of depth bombs is 9, the drop point of each depth bomb is (X_i, Y_i), and the detonation depth is d. The standard deviation of the submarine's horizontal positioning error is 120, the standard deviation of the depth positioning error is 40, the depth of the center position is 150, and the minimum depth is 120. The submarine's dimensions are length 100, width 20, and height 25. The submarine dimensions are 100 in length, 20 in width, 25 in height, and the radius of the depth bomb is 20. The probability of a single depth bomb hitting is P_d, the probability of all depth bombs failing to hit is P_m, and the probability of at least one hitting is P_x.

The probability of a single hit is:

$$\mathrm{Pd} = \sum_{i=1}^{25} P_{m_i} \tag{21}$$

The probability that all depth charges miss:

$$\mathrm{Pm} = \prod_{i=1}^{25} (1 - Pd) \tag{22}$$

The probability that at least one deep bomb hits is:

$$\mathrm{P_x} = 1 - P_m \tag{23}$$

This process is solved using MATLAB:

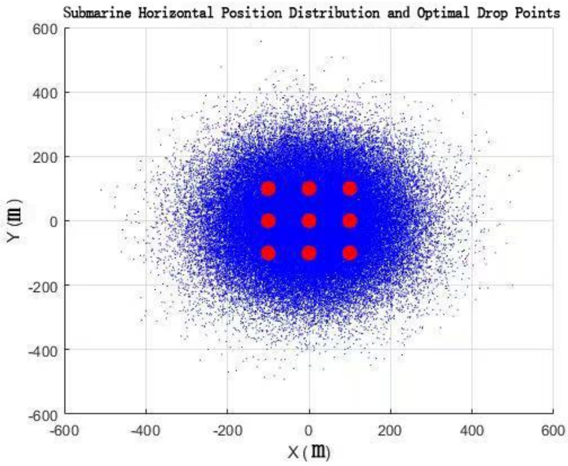


Figure 5: The best place to drop a bomb.

The Figure 5 results show that when the array layout $a = b$ is the same for the square layout, the probability of at least one deep bomb hitting the submarine is the largest. The optimal detonation depth is 125m, with $a = b = 10m$, and the maximum probability of hitting is 0.2146. The results indicate that according to the square bombing layout, throwing 9 deep bombs at the same depth simultaneously increases the hit rate.

4. Conclusions

This paper systematically discusses the strategy to enhance the hit probability of submarines and proposes a multiple deep bomb delivery optimization scheme. The model successfully maximizes the probability of hitting a submarine with at least one depth bomb by analyzing the array layout and detonation depth and combining probabilistic models with Monte Carlo methods. This scheme theoretically provides a practical bomb-dropping strategy for anti-submarine warfare. However, the model fails to comprehensively consider the dynamic changes of the ocean environment, the complex motion characteristics of submarines, and the interactions between the depth bombs during the construction process, which may significantly affect the hit probability in practice.

In order to improve the model's applicability in actual combat, it is recommended that improvements be made in the following areas: first, optimize the fuze design and explosion depth control algorithms of the depth bomb to ensure detonation at the optimal depth so as to maximize the destructive effect on submarines. Secondly, applying intelligent deep bomb technology, researching modern technologies including intelligent fuzes and adaptive explosion control, so that the deep bomb can automatically adjust the explosion parameters according to the target characteristics and environmental conditions, significantly improving the striking efficiency. Through these improvement measures, the hit rate and effectiveness of the model in practical application will be further enhanced, providing more reliable technical support for anti-submarine warfare.

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