MATLAB-based Visualization for Teaching Two Important Limits

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Abstract: Utilizing Matlab software to demonstrate and visually represent two important limits can streamline the teaching process for educators and enhance students' ability to develop three-dimensional thinking. By dynamically displaying the limit processes and supporting independent learning, this method can boost students' motivation and comprehension of abstract concepts. It encourages active participation in the learning process, providing a more efficient pathway for grasping advanced mathematical principles.

1. Introduction

Higher mathematics, as a foundational course, equips students with a robust mathematical tool for further study and research in professional fields. Despite its significance, the abstract and theoretical nature of higher mathematics complicates the teaching process, often resulting in unsatisfactory learning outcomes. Students frequently struggle to grasp fundamental concepts and theories, such as limits, derivatives, differentiations, and integrals, within a year of study. The concept of limit, serving as the cornerstone of higher mathematics poses a particular challenge for students due to its abstract nature. Understanding and applying the concept of limits ε-N, ε-δ, ε-X are critical for comprehensively learning higher mathematics. The essence of derivative, differentiation, integral, generalized integral, infinite series, and other concepts in higher mathematics fundamentally relies on the concept of limit. Given its pervasive influence throughout the higher mathematics curriculum, mastering the idea of limit is essential for students. To address the difficulties students face in comprehending limits, educators must find innovative ways to visually and vividly illustrate the limit process, fostering students' engagement and comprehension.

In today's world, as society transitions into the information age, various educational methods utilizing information technology have emerged, including distance learning and webcasting. The Matlab software, developed by MathWorks, has been particularly useful in the field of mathematics education, playing a vital role in numerical analysis, computations, and visualization research. Leveraging the numerous benefits of this software, high school students' learning efficiency can be improved, their mathematical literacy cultivated, and teachers' professionalism enhanced by integrating the software into the teaching of limits. This serves as a valuable experimental approach. Visual and graphical representations of the concept of limits in advanced mathematics are

effectively demonstrated, leading to an intuitive understanding of limit problem-solving through Matlab. This approach boosts student engagement in the classroom, increases overall teaching efficiency, grows student motivation and self-assurance in learning, and aids comprehension and mastery through a dynamic and visual teaching process[1].

Several articles focus on teaching advanced mathematics using Matlab, including text [2], which incorporates numerical calculations to aid in teaching and presents intuitive graphs for students to experiment with. Wen [3] conducted Matlab-assisted simulations of two crucial limit processes. While previous discussions have neglected visualization, it is vital in making a visual impact on students and enhancing their comprehension of mathematical concepts. A study conducted by the National Training Laboratory in Maine, USA, revealed that simply listening to lectures results in a 5% retention rate, whereas incorporating audiovisuals and demonstrations can increase retention to 50%, and hands-on practice can further boost it to 75%. Matlab offers various functions for animation, facilitating easy creation of visual aids for teaching. By combining visual and abstract thinking through visualization, students can better grasp complex mathematical concepts like limits, thereby boosting their interest and understanding. Based on the above analysis, this paper utilizes Matlab software to perform practical exercises for two important limits in higher mathematics, that is, $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim\limits_{x \to \infty} (1 + \frac{1}{x})^x = e$. We have carried out a visualization of the teaching design, the limit of the process shown to the students, the combination of image thinking and abstract thinking, not only to simplify the teacher's teaching process, but also to help students to construct the process of thinking and improve the students' learning enthusiasm, hope that this aspect of the practice to the front-line teachers to be able to provide some ideas and help.

2. Dynamic visualization using Matlab

There are two main types of Matlab animation implementations: one involves the display of dynamic graphics through the generation of frames using the geffram function and the movie command, allowing for a slight delay in presenting multiple frames per second from a human perspective [4]. The other method is real-time animation, achieved by utilizing the erase-mode attribute in graphic drawing to update only certain pixels while keeping the majority unchanged, creating a moving image. Benefiting from Matlab's integration of powerful functions in a user-friendly Windows environment, it simplifies complex tasks and is considered advanced scientific computing software, building upon the expertise of others to tackle challenging problems. Two important limits, $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim\limits_{x \to \infty} (1 + \frac{1}{x})^x = e$, are an important element of advanced mathematics. When students learn it, they feel that it is difficult and its proof process is cumbersome, and the proof process is also more complicated. With the help of Matlab programming and drawing function[5], on the one hand, we can use the command limit to calculate the limit, on the other hand, when we talk about the important limit, we can use the animation to reflect the convergence process of the limit, so that the students can understand it more easily.

2.1 Visualization $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$

We calculate the first important limit and we visualize as follows:
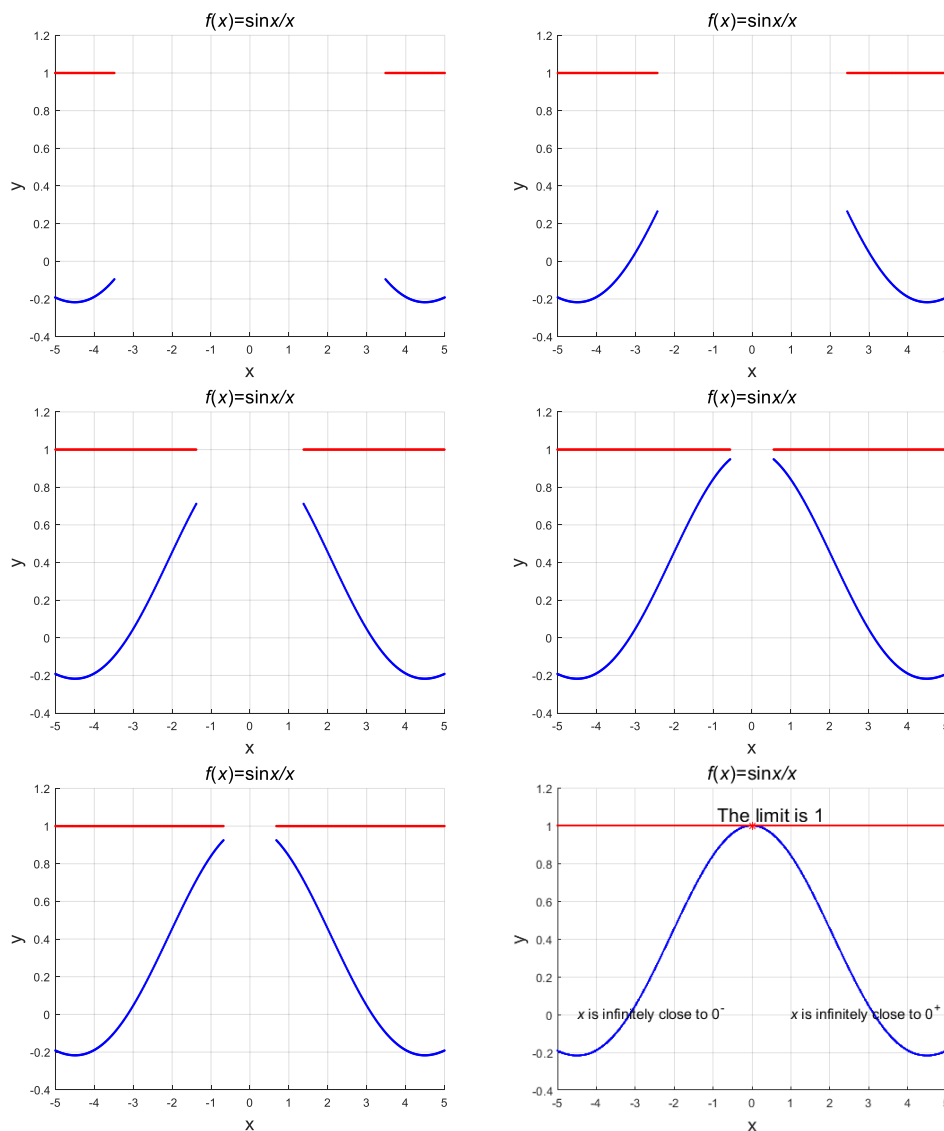
Figure 1: The Visualization $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$

As can be seen from the dynamics of Fig. 1, when x is infinitely close to the left side of 0, $\dfrac{\sin x}{x}$ is infinitely close to 1; when x is infinitely close to the left side of 0, it is also infinitely close to 1; and the closer it is to 0, the faster it tends to 1 (the steeper the curve is at this point). Figure 1 fully illustrates the limiting process.

2.2 Visualization $\lim\limits_{x \to \infty}(1 + \frac{1}{x})^x = e$

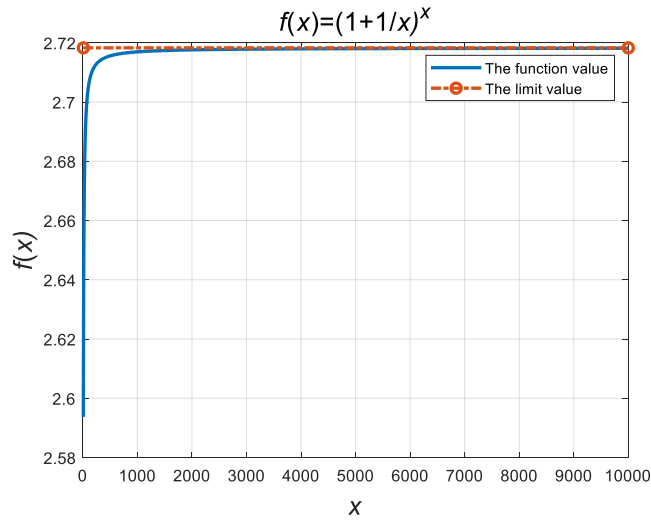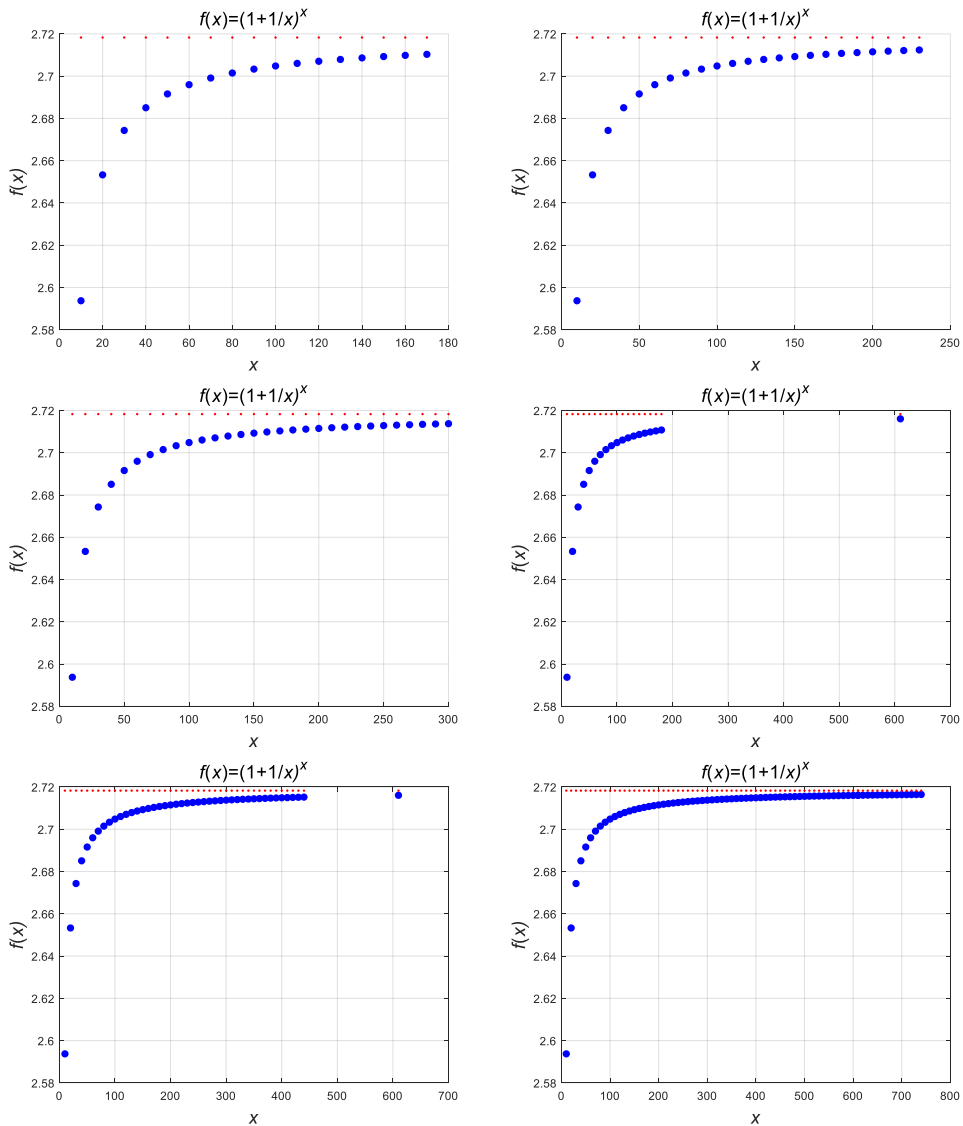Similarly, we obtain the second important limit and depict in Figure 2.

Figure 2: The Visualization $\lim_{x \to \infty}(1 + \frac{1}{x})^x = e$
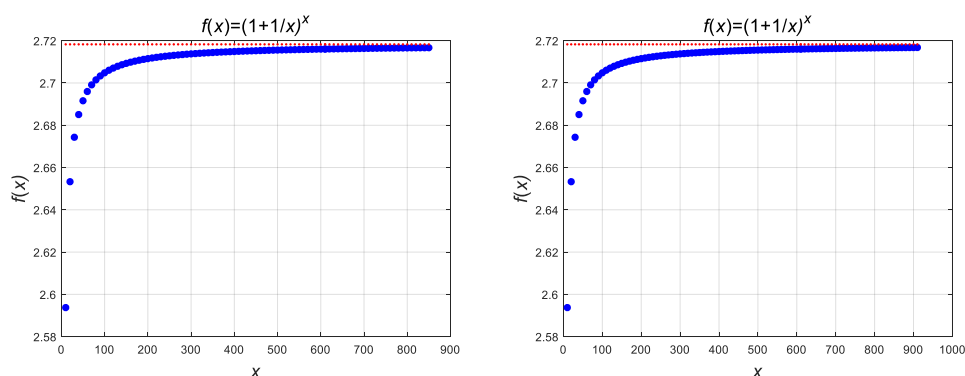
we visualize and show the results in Figure 3.

Figure 3: The Visualization $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$

From the dynamic diagram in Fig. 3, it can be seen that when x taking real numbers and converging to $+\infty$ or $-\infty$, it is infinitely close to e. The calculation result is in perfect agreement with the visualization process, which is more graphic.

3. Conclusion

When teaching advanced mathematics courses, traditional instruction tends to be more theoretical, making it challenging for students to grasp abstract concepts and lacking in spatial imagination. By leveraging the visualization capabilities of MATLAB software, dynamic visualization designs can be implemented to provide a clear and intuitive comprehension of key concepts such as limits and integrals. This approach integrates theoretical teaching with practical demonstrations, enhancing the efficiency of higher mathematics instruction, fostering students' creativity, nurturing their analytical skills, and deepening their mastery of mathematical knowledge. The use of MATLAB for visualization teaching not only facilitates students' understanding of the limit process and the significance of mathematical knowledge but also sparks their interest in mathematics. Furthermore, it lays a solid foundation for students to engage with similar computer mathematical programming software in the future, thereby motivating them to learn. The benefits of this approach extend beyond educators to positively impact the broader adoption of MATLAB, enhance students' proficiency in data analysis, and elevate the overall quality of teaching.

Acknowledgements

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