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Research and Teaching Design on Integrating Ideological and Political into Bayesian Formulas

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Abstract: At the knowledge level, students master Bayesian formulas and calculations, and are able to use Bayesian formulas to calculate posterior probabilities in practical problems. At the application level, teachers abstract real-life scenarios and cultivate students' ability to establish Bayesian models, enabling them to analyze and preliminarily solve practical problems. Students distinguish between prior and posterior probabilities and understand the widespread application of Bayesian formulas. They used MATLAB mathematical software to verify the quantitative relationship between posterior probability and prior probability. At the ideological level, through the deeds of mathematician Bayes who adhered to their original intention and pursued truth, students are cultivated with a scientific spirit of fearlessness and courage to explore. Students understand the essence and preliminary classification ideas of Bayesian formulas. We use Bayesian formula to verify the process of establishing integrity in people's minds and establish the concept of integrity. We need to strengthen students' abstract thinking and rational spirit, teachers help students establish Bayesian reasoning thinking and combine theory with practice, and help students firmly establish a concept of integrity.

1. Introduction

The inherent characteristics of Probability Theory and Mathematical Statistics [1-4] include theoretical abstraction, wide applicability, computational complexity, and dependence on computers. Although using probability and statistical methods to solve practical problems has penetrated into all fields of real life, the abstract nature of stochastic and statistical models makes them difficult for students to understand and master. In addition, the probability and statistics course takes advanced mathematics as its prerequisite, accompanied by a large number of calculus operations [5], which also makes it difficult for students to master the calculation of probability and statistics. With the rapid development of computer technology and mathematical software, opportunities and challenges have been brought to the application of mathematical statistics. Through the study of this course, students should not only master basic theories, but also have a preliminary understanding of the practical problems that probability and statistics knowledge can solve, enhance their interest in learning, and enhance their subjective initiative in learning.

This course is offered in the first and second semesters of my sophomore year at our university. During this period, students have adapted to the teaching methods of universities, possessing both

psychological conditions for learning and sufficient preparation for learning. Their comprehension and logical thinking abilities have greatly improved. They crave to have control over all aspects and the ability to independently solve problems. Therefore, teachers should pay attention to the process of student development, respect their doubts, guide them to analyze, research, and solve problems, cultivate their perseverance in learning and research, improve their research ability and mathematical literacy. At the same time, when entering the second year of college, some students have a less serious learning attitude than when they first enrolled. They start to relax in dealing with many courses and pursue passing exams, resulting in a decrease in their initiative in learning. How to motivate these students to learn is also an important task for teachers.

2. Background of Bayesian Formula [6-9]

Bayesian formula, also known as Bayesian theorem, Bayesian rule, posterior probability formula, inverse probability formula, etc., is a standard method in probability statistics that uses observed phenomena to modify subjective judgments about probability distribution (i.e. prior probability). The Bayesian formula originated from an article written by British mathematician Thomas Bayes (1702-1761) to solve an "inverse probability" problem. Prior to Bayes' article, people were able to calculate the "forward" probability, such as "assuming there are N white balls and M black balls in a bag, what is the probability of reaching in and touching the black ball?". And an inverse problem (inverse problem): "If we don't know the proportion of black and white balls in the bag beforehand, but instead touch a ball (or multiple balls) with our eyes closed and observe the color of these extracted balls, how can we deduce the proportion of black and white balls in the bag?" The prior probability is human experience knowledge, such as: if thunder strikes, it means it will rain. Winter is here, it will cool down. A posterior probability is a method of inferring the cause of an event based on known results. For example, if the dish turns sour, what may be the reason? 1: Adding vinegar, 2: It has gone bad. What is the reason for this? Bayesian formulas can provide you with solutions.

The Bayesian formula states that "by updating our initial beliefs about something with objective new information, we will obtain a new and improved belief." This research achievement, due to its simplicity, appeared mundane until two years after his death, when it was published with the help of his friend Richard Price in 1763. Its mathematical principles are easy to understand, simply put, if you see a person always doing good things, you will infer that that person is likely to be a good person. That is to say, when you cannot accurately understand the essence of a thing, you can rely on the number of events related to its specific essence to determine the probability of its essential properties. In mathematical language, it means that the more events that support a certain attribute occur, the greater the likelihood of that attribute being established. Unlike other statistical methods, Bayesian methods are based on subjective judgment. You can first estimate a value and then continuously adjust it based on objective facts.

In 1774, French mathematician Pierre Simon Laplace independently rediscovered the Bayesian formula. Laplace's concern is: how can we find the true patterns when there is a large amount of data, but there may be various errors and omissions in the data. Laplace studied the fertility ratio of boys and girls. Someone has observed that it seems that the number of births of boys is higher than that of girls. Is this hypothesis true or not? Laplace constantly collects new birth records and uses them to infer whether the original probability is accurate. Each new record reduces the range of uncertainty. Laplace gave us the expression of the Bayesian formula we are currently using: $P(A|B) = P(B|A)P(A)/P(B)$. In the formula, $P(A)$ is also called prior probability, and $P(A|B)$ is called a posterior probability.

The true attention and widespread application of Bayesian formulas have been in the past two to

three decades. The Bayesian formula began to draw attention and importance from the academic community after its remarkable performance in the case of Federalist authors and the search and rescue of the Scorpio nuclear submarine. The combination with computers further demonstrates the enormous practical value of Bayesian formulas. It not only provides us with a new path to problem-solving, but also brings about a revolution in tools and concepts. Its success in the field of natural language processing in the 1980s has shown us a brand new path to problem-solving. The continuous improvement of computing power and the emergence of big data have made its power increasingly apparent, and a magnificent Bayesian revolution is taking place.

3. The Teaching Method of Bayesian Formula

The Bayesian formula is derived based on partition, conditional probability, and total probability formulas. For students with weak logical reasoning abilities, this is an obstacle to overcome. In response to the above knowledge characteristics and student characteristics, the following measures have been taken in the teaching content, teaching methods, teaching modes, and teaching methods of Bayesian formula to improve the pertinence and practical effectiveness of teaching.

(1) Integrating ideological and political elements into teaching

To achieve the goal of imparting knowledge, cultivating abilities, and leading values in this unit, the main lecturer delves into the ideological and political elements behind the Bayesian formula. Through examples, teachers help students establish a combination of theory and practice. Teachers assist students in establishing Bayesian reverse reasoning thinking, enabling them to understand the essence of Bayesian formulas and preliminary classification ideas. By using Bayesian formula, teachers can verify the process of establishing "integrity" in people's minds and help students strengthen the concept of integrity.Through the deeds of mathematician Bayes who adhered to their original intention and pursued truth, students are cultivated with a scientific spirit of fearlessness and courage to explore.

(2)Case teaching method

Starting from the prior and posterior probabilities in the random guessing of new classmates' surnames, focus on the explanation and differentiation of conceptual nouns at the beginning of the article. By using real-life examples of disease screening and following a progressive cognitive pattern, describe the use of Bayesian formula and demonstrate the reverse thinking of Bayesian formula in "finding the cause and effect". Through the classic story "The Wolf Comes", students have been trained to use Bayesian formulas for reverse inference and firmly establish the concept of honesty and trustworthiness. Based on the problem of substandard product attribution, teachers cultivate students' Bayesian reasoning thinking to help them understand preliminary classification concepts. Throughout the entire teaching process, we continuously guide students to analyze and summarize independently, in order to upgrade their level of reverse thinking.

Teachers guide students to establish a thinking mode of "practical problems mathematical problems (posterior probability) mathematical models Bayesian models problem-solving". We start from practice, summarize theories, and finally apply them to practice. Through this teaching method, students are unconsciously trained to use their existing knowledge to solve practical problems.

(3) Project based teaching method

Teachers Designed Project -Naive Bayesian Classifier. This project integrates Bayesian formulas and independent knowledge points to form a cross fusion of probability, statistics, and classification problems. It makes the content of Bayesian formulas concrete and practical, and integrates mathematical modeling, experiments, and other forms of content to achieve modular calculation of classification problems. Teachers cultivate students' comprehensive abilities and advanced thinking in solving complex engineering problems. Teachers exercise students' programming and teamwork

abilities, cultivate their innovation awareness, help them achieve Bloom's educational and teaching goals, and enhance their application, analysis, evaluation, and creation of higher-level cognitive levels.

(4) The Application of Mathematical Modeling and Mathematical Experiments [10]

Translate practical problems such as "cancer screening", "wolf coming", and product attribution into mathematical problems, establish mathematical assumptions, and use Bayesian formulas to solve conditional probability-posterior probability. Using MATLAB experiments to demonstrate the functional relationship between prior probability and posterior probability in the case of "disease screening", so that students can fully understand the relationship between abstract prior probability and posterior probability. After class, the teacher assigns MATLAB experimental homework: Students write a MATLAB program to predict the probability of people being honest with shepherd children in the story of "The Wolf Comes". The use of mathematical modeling and experiments not only helps to arouse students' interest in learning, but also makes them realize the importance of using advanced technology to solve practical problems in the 21st century. The appropriate application of mathematical experiments in teaching can attract students' attention and arouse their curiosity, which is an important factor in stimulating learning motivation and conducting effective learning.

(5) The extraction of scientific methods

This lesson is based on partition and total probability formulas, and derives the posterior probability, also known as Bayesian formula. Teachers apply Bayesian formulas to various real-life scenarios, allowing students to have a preliminary understanding of the applications of Bayesian formulas in disease screening, psychology, classification, and more. Finally, the teacher summarizes the key points of this content to improve students' understanding ability. Teachers create scenarios of problems to stimulate students' attention, curiosity, and interest in learning, enabling them to engage in effective learning. Questions during the teaching process guide students to actively think and play a leading role in their learning.

(6) The use of mobile teaching methods

Consolidate learned knowledge through online classroom quizzes, after-school exercises, and written assignments through cloud textbooks, cloud class classes, and Maple T.A. online assessment system. The MATLAB experimental source program exercises students' hands-on ability and ability to solve practical problems. Contemporary college students, as electronic natives, have a strong interest in online activities. Online tests and assignments can quickly attract their attention. More importantly, they can enable teachers to quickly and timely understand students' mastery of the knowledge points they have learned, and conduct timely teaching evaluations and reflections.

4. Teaching design

In teaching, the teacher is the leader, and the students are the main body. All teaching activities should fully mobilize the enthusiasm, initiative, and creativity of students in learning.

Teachers upload micro videos of knowledge points to the cloud classroom in advance for students to preview and complete their homework. Students should have an early understanding of prior probability, posterior probability, Bayesian formulas, and their applications. In the classroom, teachers and students only focus on solving difficult problems. In this lesson, the main teacher combines "random guessing of new classmates' surnames" to clarify the connection and difference between prior probability and posterior probability, and starts with a topic to attract students' interest in learning. Next question: How is the posterior probability calculated? Who discovered this probability in the history of mathematics? (1) Enable students to understand what prior probability and posterior probability are and their differences; (2) Introduce the history of mathematics. The

teacher deepens students' impression of the widespread application of Bayesian formulas and stimulates their interest in learning by introducing a simple history of mathematics.

The teacher will use the courseware in conjunction with the blackboard to introduce the derivation conditions of Bayesian formula. The teacher leads students to review old knowledge -partitioning, conditional probability formulas, multiplication formulas, total probability formulas, and deduces Bayesian formulas. By continuously interweaving and using old knowledge, new knowledge can be derived to enhance students' impression and logical reasoning ability towards conclusions and their conditions of validity. We need to explain the meanings of prior probability and posterior probability in Bayesian formulas, what are "cause" and "effect", and why Bayesian formulas are called inverse probability formulas. By explaining new terms and meanings in the formula, teachers help students understand the formula and establish a preliminary reverse thinking method for Bayesian formulas.

Teachers describe the use of Bayesian formula and reverse thinking in Bayesian formula by introducing real-life examples - cancer screening, following a progressive cognitive pattern. Firstly, the teacher transforms the actual problem of cancer screening into a mathematical problem and guides students to make model assumptions. The teacher explains the meaning of the probability corresponding to the hypothesis, pointing out the "cause", "result", prior probability, and posterior probability. Teachers enhance students' mathematical awareness and thinking, assist them in using mathematics to observe and explain various phenomena in life, and help students establish Bayesian reasoning thinking. The teacher uses the blackboard to solve the calculation of conditional probabilities based on Bayesian formulas with students, and compares the sizes of prior and posterior probabilities; The teacher explains the significance of posterior probability size. The teacher explains the importance of cancer screening and guides students to think rationally. By using Bayesian formula for the first time, the probability of developing cancer under a positive reaction is obtained, and the different meanings of posterior probability for patients and doctors are pointed out. If the test result is positive, avoid panic, follow the doctor's advice, and have a follow-up examination as soon as possible. Secondly, taking the first posterior probability as the prior probability for reexamination, and keeping all other conditions unchanged, continue to use the Bayesian formula to obtain the second posterior probability, which is close to 0.7, indicating the necessity of reexamination work. Next, in order to take responsibility for the patient, a comprehensive physical examination will continue. On the second follow-up, if the result is still positive, the posterior probability is almost 1, and the patient can be basically diagnosed with cancer. When explaining the continuous conversion between posterior probability and prior probability, doctors should make reasonable diagnoses and patients should make correct decisions, explain the importance of reexamination, guide students to think rationally, and guide their future lives. The Bayesian formula solves the conditional probability of "a known result occurring and a certain reason occurring", which belongs to the process of "finding the cause through action" and is a kind of reverse thinking. Teachers use MATLAB programs to analyze the quantitative relationship between prior probability and posterior probability. Teachers use program images to help students visualize the relationship between prior and posterior probabilities. Finally, if all other conditions remain unchanged, calculate the posterior probability (if the test result is negative, the probability of the patient being cancer) using Bayesian formula as 0.0002, which is extremely low. A negative reaction indicates that there is no need for a follow-up examination. Teachers encourage students to solve abstract mathematical problems and concrete practical problems through programming, strengthen their Bayesian reasoning thinking, show students that Bayesian formulas can guide our lives, and help students establish reverse thinking of Bayesian formulas.

Teacher reviews the story - the wolf has come. The teacher helps students adapt the story of the wolf into a mathematical problem and make reasonable assumptions. Teachers and students work

together to establish Bayesian models to discuss the changes in credibility of children after lying. The teacher introduces the application of Bayesian formula in psychology. The student used Bayesian formula twice and gradually learned that after the first time the shepherd child lied, the probability of people's trust in him decreased from 0.7 to 0.318. After the second time the shepherd boy lied, the probability of people's trust in him decreased to 0.085, less than 10%. People believed that the child was a "lying" child, so the third time the shepherd boy called the wolf again, and people already believed that he was "lying" and no longer saved him. The decision-making process of people reflects the continuous improvement of their understanding of shepherd children. Teachers stimulate students' interest in learning Bayesian formulas through familiar stories. Throughout the entire reasoning process, the teacher not only exercised students' thinking of using Bayesian formulas for reasoning, but also enhanced their belief in honesty and trustworthiness.

We will apply Bayesian formula to the problem of attribution of defective products in the product. By addressing the issue of the attribution of defective products, we aim to stimulate students' interest in learning Bayesian formulas and enable them to experience the application of Bayesian formulas in classification problems. Students can identify the "cause" and "outcome", and use the meaning of the problem to analyze various possibilities and conditional probabilities. Students use the full probability formula to calculate the probability of a result occurring, i.e. the defect rate, allowing them to experience the application of the full probability formula. We will use Bayesian formula to determine the probability of defective products being produced by the first manufacturer, and ask the probability of defective products being produced by the second or third manufacturer. Teachers inspire students to think deeply and help them solve the problem of product ownership. Teachers help students experience the application of Bayesian formulas in classification problems and strengthen their ability to establish Bayesian reasoning thinking. Students summarize the general steps and process of using the total probability formula and Bayesian formula to solve problems on their own; Students write the problem-solving process on a blackboard or notebook. The teacher checks whether the students have learned how to calculate the total probability formula and Bayesian formula. Teachers should promptly check the students' mastery of the knowledge they have learned and identify and fill any gaps. Bayesian formula is applied to processing spam, classifiers, searching for missing nuclear submarines, artificial intelligence, etc., expanding students' knowledge and emphasizing the universality of Bayesian formula application. The teacher summarizes the new knowledge learned in this lesson, so that students have a clear understanding of the main learning content of this lesson. Assignment: (1) Paper assignments are two practical questions that test students' basic computational and practical abilities; (2) Expansion homework is the application of Bayesian formula in the reliability of sheep herding dolls using MATLAB software; (3) Reflection homework is a famous "three door problem" that is solved using Bayesian formulas or other methods and submitted in small groups. Paper assignments mainly check the writing standards of students, cultivate their rigorous logical reasoning and calculation abilities, and establish good writing habits; We need to deepen students' understanding of the knowledge they have learned and exercise their ability to apply it through expanding and thinking about homework and prepare the homework for the next course to learn independence.

The teacher emphasized the key points of this lesson and made analogies between new and old knowledge to inspire students to think actively. Teachers use methods such as narration and questioning to test students' mastery and cognitive level in the classroom. Some students use the blackboard for classroom exercises, while others upload their classroom exercises to the cloud class to test their learning effectiveness in a timely manner.

5. Teaching reflection

In terms of teaching design, this course starts from the real-life scenario of "guessing the surnames of new classmates", analyzes and summarizes mathematical problems, and ultimately solves practical cases through learning mathematical theoretical knowledge. This design is conducive to enhancing students' interest in learning knowledge points and cultivating their mathematical modeling abilities. From the on-site perspective, students have a relatively high acceptance rate.

In the teaching process, some heuristic language is used to guide students to think together, extract basic problems, and reproduce the path of brain intuitive judgment. On site students can actively explore based on the guidance of the teacher. Interpret disease screening in real-life scenarios through new knowledge and cultivate students' ability to solve practical problems. By using familiar stories to visualize abstract concepts, students can fully understand the knowledge points and grasp the essence of Bayesian formulas. Through appropriate knowledge extension, students can understand the widespread application of the knowledge points they have learned, broaden their thinking, and stimulate their enthusiasm to apply the knowledge they have learned to practice. Full of spirit and passion, it effectively stimulates students' enthusiasm for learning. Teachers should use MATLAB programs appropriately to make abstract content more specific and easily stimulate students' hands-on abilities.

Existing problems and improvement measures: (1) During the teaching process, it was found that some students did not have a good grasp of which random event was the cause and which random event was the result. In future teaching, it is necessary to strengthen the explanation and practice of this part of the content. (2) The selection of cases should take into account both universality and novelty, in order to better attract the learning interests of students from different fields or majors.

6. Conclusion

Teachers use case teaching methods to showcase learning content and stimulate students' interest in learning. Teachers focus on guiding students to think through heuristic and questioning language. Teachers highlight the connection and difference between new and old knowledge, highlighting key points. The teacher combines blackboard writing with PPT, demonstrates the process through multimedia, and summarizes the key content on the blackboard. Teachers use MATLAB software to simulate the relationship between prior probability and posterior probability, providing students with an intuitive experience. Teachers conduct classroom tests through classroom exercises, strengthen post class exercises based on cloud textbooks, cloud classrooms, and homework, and promptly identify student problems and make up for them. Teachers use Bayesian formulas to solve practical problems, guide students in Bayesian reasoning, and help students establish reverse thinking. Taking students as the center, teachers adopt a teaching model of "problem introduction - inspiring thinking - analyzing reasoning - building knowledge - solving problems" to encourage students to learn Bayesian formulas.

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