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Simulation to Washing Processes and Optimal Strategy for Using Laundry Detergents: A Concise Mathematical Approach

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Abstract: Generally speaking, none of us want to keep our clothes with dirt. One of things people do on a daily basis is to wash their clothes and dry them. Considering that the amount of fresh water is limited and the abuse of detergents may cause environmental pollution, this article simulates the general process of washing clothes using a washing machine by constructing a corresponding mathematical model. This article considers the different washing times and the changing trend of the proportion of stains in the clothes-water-detergent-dirt system under different initial amounts of stains, different detergent usage, and different washing times, based on the assumption that there are certain rules for the solubility of detergents. At the same time, it collects some information about the solubility of stains and detergents, and proposes a solution that minimizes the total cost through a 0-1 integer programming model. This research will probably benefit laundry while saving both water and energy.

1. Introduction

Laundry cleaning is something people do on a daily basis. The decontamination function of laundry detergent comes from some surfactant chemicals, which can increase water's permeability and utilize the intermolecular electrostatic repulsion mechanism to remove dirt particles. Due to the form of the surfactant molecules, laundry detergent can exert its dual function. One end of the surfactant molecule is lipophilic, which attracts dirt and repels water, while the other end is hydrophilic, attracting water molecules. When the laundry detergent is poured into water, the lipophilic part of the surfactant molecule vigorously adheres to any surface not filled with water, such as the object's surface being washed (due to the body or fabric surface). At the same time, the hydrophilic part repels oily substances, weakening the intermolecular forces that maintain the binding of water molecules (the forces that cause water to form droplets like being wrapped in an elastic film), allowing individual molecules to penetrate between the surface of the object to be cleaned and the dirt particles. Therefore, it can be said that surfactants reduce the surface tension of water. The mechanical action of a washing machine or hand rubbing can cause the dirt particles surrounded by surfactant molecules on the surface to become dislodged, and the dirt particles adhere to the lipophilic portion of the surfactant molecules. This causes the dirt particles still suspended on the object's surface to be removed during the Rinsing phase.

In practice, from small-scale household operations to professional operations in hotels and specialized institutions, the question that needs to be considered is: How can we get the laundry clean and tidy at a lower cost? This thesis offered a simulation to the processes of washing processes, helping to reduce not only more water but also more time to wait.

All the price units involved in this article are "yuan", which is the unit of the legal currency of the People's Republic of China, Renminbi. The city where the author works and lives is Suzhou, a city in Jiangsu province, China. This unit of currency is generally used in Suzhou, together with ‘liter’, an another commonly used unit of volume, with 1 liter equal to 0.1 cubic meter. "Gram" is a commonly used unit of weight in Suzhou, with 1 gram equal to one thousandth of a kilogram.

2. Assumptions

2.1. Assumption 1

The solubility of dirt in water changes according to a specific rule. Different temperatures have no affect to the solubility of dirt. The differences between different kinds of dirt are ignored, except different solubility of dirt in different kinds of detergents.[1][2]

2.2. Assumption 2

Only conventional water-washing methods is only considered.

2.3. Assumption 3

The detergent effect is mainly determined by the dirt solubility.

2.4. Assumption 4

The mode of the washing machine stays unchanged.

2.5. Assumption 5

Parameters describing the working status of the washing machine all stay unchanged during the process considered in the simulation and modelling.

2.6. Assumption 6

We globally considering the clothes-water-detergent-dirt system. In the system we consider, no water will be removed out of it.

3. An concise mathematical description and simulation to washing processes

Consider the simplified processing of washing processes, we use mathematical approaches to describe it. Initially, we define variables as Table 1:

Table 1: Symbols and Meanings for Mathematical Modelling Describing Basic Mechanism

Symbols	Meanings
N	times of washing
V_k	water volume used for the k-th washing
C_k	the residual amount of dirt after the k-th washing
a_k	the given solubility
C_0	the amount of dirt initially
W	The maximum amount of available water

The initial dirt amount: represents the quantity of dirt on the clothing before cleaning, measured in grams. The amount of dirt residue after each wash can be calculated using the following recursive relationship($k = 1,2, \ldots$):

$$C_{k+1} = a_k C_k \tag{1}$$

In this mathematical model, $a_k$represents the given solubility, as we assume in the question with $a_1 = 0.80$ and $a_k = 0.5a_{k-1}$.[3]

Considering the limited amount of available water, we can introduce a constraint to ensure that the total water consumption does not exceed the available water in the target, namely:

$$\sum_{k=1}^{N} V_k \leq W \tag{2}$$

Our goal is to determine minimal amount of washing N and ensuring the cleaning effect is fit for the requirements listed in Table 2.

Table 2: Initial values for the simulation

Symbols	Values	Symbols	Values
C_0	100	W	200

We have gained that optimal $N = 3$ in this situation.

4. Time-efficient cleaning plan and analysis

We define variables like that in Table 1. And our solution goal is to minimize the whole time consumed. The constrains include:

(1) The residual amount of dirt after each washing shall not exceed one thousandth of the initial amount of dirt. Namely:

$$C_L = \frac{1}{1000} C_0 \tag{3}$$

(2)Water amount is a non-negative real number. Namely:

$$V_k \geq 0 \tag{4}$$

The rest part of the model (including the constrains of water available and amount of dirt initially) is same as previously modeled. Simulate the whole process, we drew the curve as Figure 1:

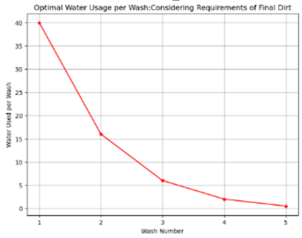

Figure 1: Optimal Water Usage per Wash(Considering Requirements of Final Dirt)

The water usage strategy (optimal) is shown in Table 3:

Table 3: Optimal Water Used per Wash

Wash-rows	Optimal water used per wash
1st	40
2nd	16
3rd	6
4th	2
5th	0.5

It has been known from calculating that optimized $N = 5$ in this situation.

5. Sensibility Analysis

In this part, the impact ofa_k, the initial dirt amount and restrictions of water resources available on the optimal solution are considered separately. Furthermore, we assume that there are no constrains of water available, initially. The more the clothes is washed, the more water the whole process will cost. And we assume that each time of washing costs 100 liters of water. Dust content is here defined to measure the process of detergent. [3][4]

5.1. Research on impact of a_k.

In recursive expression

$$a_k = pa_{k-1} \tag{5}$$

We assume that $0 < p < 1$.As for all k there is $a_k \geq 0$,it is easy to prove that

$$\lim_{m\to\infty} a_m = 0 \tag{6}$$

For different values of $0 < p < 1$,the amount of dirt residue after each wash, describing as:

$$C_{k+1} = a_k C_k \tag{7}$$

has different convergence rates.

Using different values p when simulating the whole processes, results are listed in Table 4:

Table 4: Values of ps and their times to wash (simulating)

Values of p	Times to Wash	Values of p	Times to Wash
0.1	5	0.5	13
0.15	6	0.6	17
0.25	7	0.75	24
0.3	8	0.8	26
0.45	12	0.9	17

Here we all fix $a_0 = 0.8$, assuming that the amount of dirt on the clothes stays at 10 grams. Simulation results can be seen in Figure 2(a)~2(c).

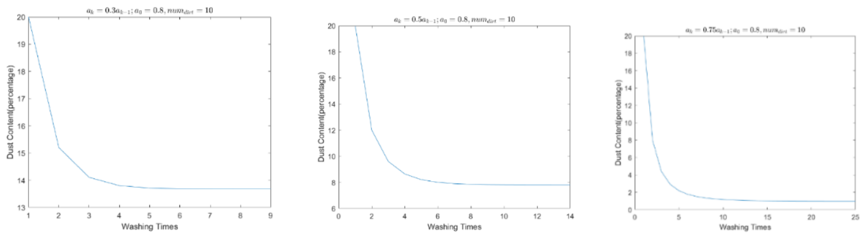

Figure 2(a)~2(c): Dust Content(perentage)-Washing Times at $p = 0.3,0.5,0.75$

From the calculation results(especially from the curves),we have known that when p increases on open interval $(0,1)$,although it will take (approximately) same time to wash ,the dust content converge at rates from faster to lower. In reality, different detergents represent different values of p, and in reality their ability to bring dust out of the clothing fabric. The smaller p is, the better ability the detergent have, and it may require water to wash both detergent and dirt away.

5.2. Research on initial dirt amount on the optimal solution

In following research, to show how will amount of initial dirt make difference to the optimal solution, we will fix p, only changing initial values of the amount of dust, varying from 5 grams to 20 grams initially.

The simulation results can be seen as Table 5 shows:

Table 5: Simulation of Washing Times with Different Values of Initial Amount of Dirt

Values of num_{dirt}	Washing Times	Values of num_{dirt}	Washing Times
2.5	11	20	14
5	12	25	14
10	13	50	15
15	14	100	16

The more dirt on the clothes, the more times it will take to wash by hands or washing machine. The results of simulation cooperates with our daily life experiences. Figure 3(a)~3(c) describe the process in a visualized method.

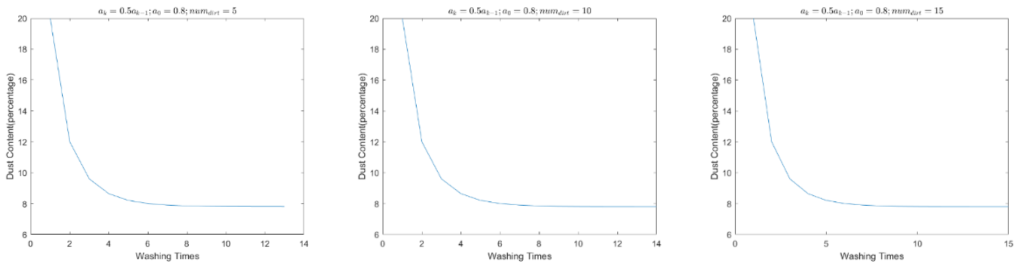

Figure 3(a)~3(c): Dust Content(percentage)-Washing Times at $num_{dirt} = 5,10,15$

5.3. Analysis on restrictions of water resources

Like what has done before, fix other parameters and their values as Table 6:

Table 6: Parameters Fixed in Research of Restrictions on Water Resources

Parameters	Values
a_0	0.8
Value of p	0.5
Amount of dirt initially	10

Fixing these parameters above, and the simulation results are given in Table 7:

Table 7: Simulated Times to Wash Considering Different Restrictions on Water Resources

Restrictions on water	Times to wash	Restrictions on water	Times to wash
10^2	No Feasible Solutions	10^4	13
10^3	No Feasible Solutions	10^5	13

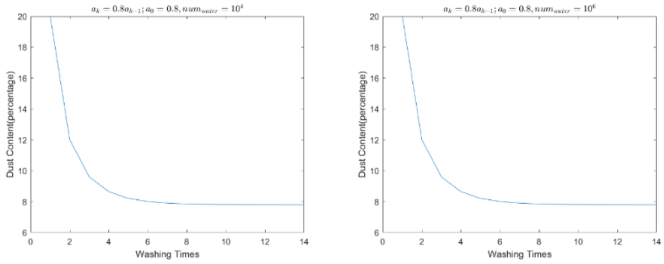

Figure 4(a)~4(b): Dust Content(percentage)-Washing Times at $max_{water} = 10^4, 10^5$

Comparing figures 4(a) and 4(b), it is clearly that if the water available is adequate to use, more water will not lead to less times of wash .It suggests that we could make assumptions that the water is full-adequate to use and deciding the least times of washing. When the least times of washing is calculated (assuming that the water consumption during a single washing does not change), it is evident to calculate the least water usage results.

6. A cleaning scheme considering mixed-washing situation

In real life, we usually wash clothes after classifying them. Clothes under the same classification standard are often washed at the same time and in the same washing machine. Different clothes may have different stains, and the solubility of different detergents for different stains may not be the same. Chemical reactions may occur between different detergents or between specific detergents and specific materials of clothes, which may affect the wearing of clothes after washing. These all put forward new requirements for the optimal use of detergents in real life. Based on the previous article, further research is conducted on the compatibility of "parallel imports" detergents and between detergents, as well as between detergents and clothes. Assume that there are no limits to the amount of available water.

We define another variables as Table 8: [1][2][4]

Table 8: Symbols and Meanings for Mathematical Modelling about Mixed-Washing Scene

Symbols	Meanings
x_i	Quantity of detergent i
y_j	Total amount of dirt type j
c_i	Unit price of detergent i
D_{max}	Available amount of detergent
W_s	Available amount of water
W_u	Water usage
W_{cr}	Water cost rate
D_i	Amount of i-th detergent used
G_j	Dirt amount on i-th clothing
R_j	Dirt residue after j-th washing
ϕ_k	Boolean variables deciding whether garment k could be washed or not
m_k	Noting the material type of each garments

Hence, the aim is to minimize the total cost, including the cost of detergent and water, namely:

$$minimize\ W_s + \sum_{i=1}^{10} c_i x_i \tag{8}$$

The cost of water can be expressed as :

$$W_s = W_u W_{cr} \tag{9}$$

Constrains:

(1) The total quantity of each type of dirt cleaned by each detergent should meet the needs of dirt:

$$\sum_{i=1}^{10} D_i x_i \geq G_j, j = 1,2, \dots ,8 \tag{10}$$

(2) The use quantity of each detergent should be non negative.

$$x_i \geq 0, i = 1,2, \dots ,10 \tag{11}$$

(3) The use quantity of each detergent should not exceed the inventory.

$$x_i \leq D_{max} \tag{12}$$

(4) The water volume shall meet the requirements of washing

$$W_u = \sum_{i=1}^{10} x_i \tag{13}$$

(5) The dirt residue after washing shall not exceed one thousandth of the initial amount of dirt.

$$\sum_{i=1}^{10} D_i x_i R_j \leq \frac{1}{1000} G_j, j = 1,2, \dots ,8 \tag{14}$$

(6)To describe the feasibility between detergents and clothes, a set of Boolean variables are introduced:

$$\varphi_k = 0 \; or \; \varphi_k = 1, k = 1,2, \dots ,10 \tag{15}$$

If $\varphi_k = 0$, then:$\sum_{i=1}^{10} D_i x_i m_k = 0$.If $\varphi_k = 1$,then:$\sum_{i=1}^{10} D_i x_i m_k \geq 1$.

We have gathered information including these data:

(a)Types and quantities of contaminants on each garment. It includes 36 types of garment and 8 contaminants. Table 9 offered the types and quantities of contaminants on each garment.

Table 9: Types and Quantities of Contaminants on Each Garment (Unit: grams)

Garment	Contaminant 1	Contaminant 2	Contaminant 3	Contaminant 4	Contaminant 5	Contaminant 6	Contaminant 7	Contaminant 8
1	8	5	2	4	3	1	0	0
2	3	2	4	5	0	0	0	1
3	2	8	2	1	2	2	1	0
4	2	0	5	3	2	1	4	3
5	2	0	4	7	1	3	4	2
6	0	0	0	6	5	4	0	0
7	2	7	4	5	2	2	5	1
8	1	0	4	3	6	5	2	0
9	7	2	0	5	3	1	0	4
10	5	6	4	2	2	2	4	4
11	5	0	5	0	2	4	2	4
12	6	2	8	3	4	0	2	3
13	0	6	5	4	2	2	3	1
14	5	2	3	1	2	3	4	0
15	3	0	0	0	4	5	7	6
16	0	0	0	4	0	5	6	9
17	5	6	0	5	2	4	1	1
18	5	1	2	3	4	0	4	2
19	1	7	8	2	3	0	0	4
20	2	0	1	0	4	3	0	0
21	6	5	4	0	2	3	1	1
22	5	2	0	4	5	1	3	2
23	0	4	1	5	1	0	3	3
24	1	1	2	5	3	0	4	3
25	5	0	0	5	6	3	1	2
26	1	1	3	3	0	0	4	2
27	9	2	3	2	5	6	1	2
28	6	2	4	2	0	0	4	3
29	0	3	4	0	0	0	0	0
30	7	5	0	0	5	2	2	0
31	5	2	3	3	7	0	1	1
32	4	2	0	3	1	0	2	1
33	0	0	2	4	6	0	1	0
34	0	5	2	0	1	0	5	1
35	2	1	2	4	0	3	4	6
36	1	2	5	3	4	6	1	4

(b)Solubility of various detergents on the dirt and unit prices of detergent. Shown in Table 10.

Table 10: Solubility of Contaminants for Various Detergents and Unit Price of Detergents (Unit: yuan/gram)

Detergent	Contaminant 1	Contaminant 2	Contaminant 3	Contaminant 4	Contaminant 5	Contaminant 6	Contaminant 7	Contaminant 8	Unit Price
1	0.54	0.75	0.78	0.69	0.8	0.66	0.55	0.73	0.25
2	0.77	0.67	0.59	0.58	0.71	0.48	0.45	0.66	0.09
3	0.7	0.64	0.62	0.71	0.63	0.78	0.5	0.8	0.11
4	0.51	0.54	0.66	0.82	0.7	0.6	0.46	0.55	0.19
5	0.39	0.72	0.43	0.57	0.46	0.53	0.4	0.6	0.08
6	0.45	0.6	0.53	0.48	0.55	0.45	0.49	0.77	0.1
7	0.69	0.55	0.62	0.56	0.47	0.38	0.44	0.64	0.12
8	0.52	0.44	0.63	0.71	0.56	0.68	0.36	0.66	0.11
9	0.8	0.65	0.56	0.49	0.73	0.55	0.47	0.61	0.18
10	0.47	0.81	0.77	0.53	0.64	0.59	0.53	0.42	0.22

Table 11: Optimal Price Calculated with Clothes described in Table 9

Number	1	2	3	4	5	6
Cost	3.24	2.30	2.81	3.40	4.04	2.50
Number	7	8	9	10	11	12
Cost	4.69	3.77	3.49	4.88	3.91	4.65
Number	13	14	15	16	17	18
Cost	3.88	3.83	4.67	4.11	4.09	4.00
Number	19	20	21	22	23	24
Cost	4.26	2.14	4.14	4.19	2.98	3.62
Number	25	26	27	28	29	30
Cost	4.24	2.88	5.71	4.27	1.60	4.19
Number	31	32	33	34	35	36
Cost	4.43	2.78	2.80	2.88	4.21	4.86

Table 11 gives the optimal cost allocated by each garment(unit: yuan).It suggests that probably at optimal situation, all kinds of clothes will not destroyed by certain kind of detergents by possible chemical reactions happening during the washing processes.

7. Feasibility Analysis

7.1. On mathematical model itself

(1) Dirt solubility assumption: the model assumes that the solubility of dirt in water changes according to a specific rule. This assumption may be affected by different detergent and dirt types in practice. [1][3]

(2) Detergent effect assumption: the model assumes that the detergent effect is mainly determined by the dirt solubility, while other factors (such as detergent composition, colour protection, etc.) are ignored in the mathematical model.[2]

7.2. On complexity of actual situation

(1) Types and materials of clothes: the scheme considers clothes of different materials, but in practice it may also involve more special situations, such as special materials or processing requirements. [3][4]

(2) Other factors: the model does not take into account the applicability of detergent ingredients to different clothes, as well as other possible factors in the washing process (such as washing machine model, temperature, etc.)[4][5]

7.3. Feasibility considerations

(1) Feasibility of actual operation: the optimal solutions of cleaning times, water volume and detergent consumption are mentioned in the scheme. Whether these values are within the operable range should be considered in actual operation. [2][5][4]

(2) Detergent and water cost: the scheme minimizes the cost, but in practice, the supply of detergent and the payment of water cost should be considered. Water prices in different regions are often different, different brands of detergents have different effects on cleaning the dirt and solubility.

8. Conclusions and Extensions

The thesis has finished simulation of washing processes when we use washing machine. It also considers the different washing times and the changing trend of the proportion of stains in the clothes-water-detergent-dirt system under different initial amounts of stains, different detergent usage, and different washing times, based on the assumption that there are certain rules for the solubility of detergents. At the same time, some information about the solubility of stains and detergents are collected, proposing a solution that minimizes the total cost by water and detergent through a 0-1 integer programming model.

Available fresh water always holds a small part of water resources on the Earth. None of us could live without adequate amount of water. Water wasting and pollution could be somehow more reduced by relevant work around the washing processes. Doing more experiments on different kinds of dirt and their solubility in various kinds of detergents is undoubtedly a must. It is also a good way to discover more about details of mathematical models. We can furthermore consider the regular describing the solution process of dirt, thus enabling a more accurate approach.

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