

Analysis of Fungal Decomposition Wood Based on Logistics-Volterra Model

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Abstract: As an important component of the geochemical cycle, carbon cycle is mainly composed of the decomposition of plant material and woody fibers, which is mainly completed by fungi. The traits of fungi and the interaction between various fungi will affect the decomposition rate, and then affect the process of carbon cycle. Therefore, it is of great significance to study the decomposition process of ground litter and wood fibers. Based on Logistic Model and Volterra Model, this paper mainly studied the relationship between the decomposition process of ground litter and woody fibers and the traits of fungi, environment and biodiversity. The decomposition rate of fungi to ground litter and woody fibers is related to some traits of fungi. We mainly focus on the growth rate and moisture tolerance of fungi. On the basis of Volterra Model, we add the logistic term, building the Logistic- Volterra Model. At the same time, we also introduce environmental temperature and humidity into the model. In the model, the decomposition ability of fungi to ground litter and woody fibers is related to temperature, and the death rate of fungi is related to humidity. According to the model, we analyzed the change trend of the mass of ground litter and woody fibers and the overall length of hyphae with time.

1. Introduction

Carbon cycle, which refers to the phenomenon on the earth that carbon exchange during the biosphere, lithosphere, hydrosphere and atmosphere, and circulate constantly with the rotation of the earth. The basic process of carbon cycle is as follows: the carbon dioxide in the atmosphere is absorbed by plants on land or in the ocean, and then return to the atmosphere in the form of carbon dioxide by biological or geological processes and human activities. In the atmosphere, carbon dioxide is a crucial gas in the material cycle. The carbon cycle is the material cycle chain between the inorganic environment and organic organisms, which maintains the balance of CO₂ in the atmosphere.[1]

Soil fungi, which account for 81 to 95 percent of the overall microorganisms, not only does them promote photosynthesis, but also are responsible for decomposing organic matter and releasing them into the atmosphere in the form of carbon dioxide. Fungi play an important role in the plant-soil-atmosphere carbon cycle.[2] For the progress and development of human beings as well as for the further understanding of the biological regulation mechanism in the global carbon cycle, It is of great significance to acknowledge the role of fungi in the terrestrial ecosystem carbon cycle. At the

same time, in order to provide a reference for the research of increasing carbon storage in the ecosystem and reducing atmospheric CO₂ concentration, in this paper, we will specifically discuss the role of fungi in the ecosystem.

In this paper, a mathematical model is established to analyze the decomposition rate of wood fiber and ground dropped material, and the quantity of these materials changing with time is obtained. At the same time, the effects of different temperature and moisture resistance on the decomposition rate were investigated. The effects of atmosphere and climate change on the decomposition of wood fiber and ground debris by fungi were investigated. In this paper, a mathematical model is established to analyze the decomposition rate of wood fiber and ground dropped material, and the quantity of these materials changing with time is obtained. At the same time, the effects of different temperature and moisture resistance on the decomposition rate were investigated. The effects of atmosphere and climate change on the decomposition of wood fiber and ground debris by fungi were investigated.

2. Volterra-Logistic Model

After a comprehensive analysis of the problem, to simplify our model, we make the following reasonable assumptions:

- (1) We suppose that the quantity of woody fibers and ground litter will not increase.
- (2) We assume that the effects of water and heat generated during the decomposition of woody fibers and ground litter on the environment of the fungus are negligible.
- (3) It is assumed that the temperature studied in the experiment will not kill the fungi or let the enzyme inactivation. It is believed that when the temperature range from 10°C to 20°C, the higher the temperature is, the faster the decomposition rate will be.

2.1. Background of the study area

Before studying fungi activities in the existence of multiple fungi in the decomposition of ground litter and woody fibers, we firstly study the existence of a single fungus. Because of the limited environmental resources, the number growth of fungi meets the Logistic Growth Model [3], it should meet the equation 1. As stated above, we suppose that the mass of wood fiber or ground litter is x , the overall length of hyphae is y , the growth rate of woody fiber under laboratory conditions is r_1 , the growth rate of fungal is r_2 , the environmental carrying capacity is N .

$$\frac{dy}{dt} = r_2 y \left(1 - \frac{y}{N}\right) \quad (1)$$

We fix r_2 as 0.2 and N as 50, it can be seen from the Figure 1:

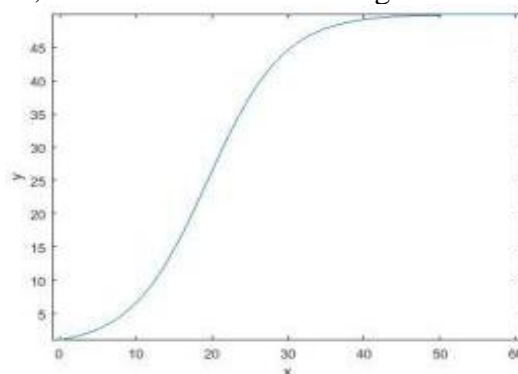


Figure 1: Logistic Model

2.2. The Establishment of Volterra Model

As fungi decompose leaf litter and woody fibers, the mass of these things affects the hyphal extension rate. Meanwhile, the growth of hyphae will also influence the decomposition rate of fungi. Hence, we build the Volterra model to investigate the correlations between woody fiber decomposition and fungi. From what we discussed above, we assume that the support capacity from woody fibers or ground litter to fungi is b , the death rate of fungi is d , the environmental temperature is T , the moisture tolerance is q and the ability of fungi to decompose woody fibers or ground litter is a . Among that, a is related to temperature, we denote that the correlation coefficient is k_1 . The death rate of fungi is relevant to moisture tolerance and environmental humidity, and the environmental humidity is set as k_2 . On the basis of this model, we can obtain the equation as follows:

$$\begin{cases} \frac{dx}{dt} = x(r_1 - ay) \\ \frac{dy}{dt} = r_2 y \\ r_2 = bx - d \\ a = k_1 T \\ d = \frac{k_2(1-q)}{2} \end{cases} \quad (2)$$

2.3. The Establishment of Volterra-Logistic Model

Via combining the Logistic Growth Model and Volterra Model, we can make a correction and get the Volterra-Logistic model. This model reflects the relationship between decomposition rate and feeding capacity, death rate as well as hyphal extension rate, the equation should meet:

$$\begin{cases} \frac{dx}{dt} = x(r_1 - ay) \\ \frac{dy}{dt} = (bx - d)(1 - \frac{y}{N}) y \end{cases} \quad (3)$$

We expand the equation (3) to the equation (4)

$$\begin{cases} \frac{dx}{dt} = x(r_1 - k_1 T y) \\ \frac{dy}{dt} = (bx - \frac{k_2(1-q)}{2})(1 - \frac{y}{N}) y \end{cases} \quad (4)$$

2.4. Solution of the Model

By collecting datas come from reference, we obtain the relevant information about 34 species of fungi. Among that, some of the fungal information as follows Table 1-3 [4].

Table 1: Decomposition rate measured for each isolate at 10, 16 and 22°C

Isolate	10 °C	16 °C	22 °C	geometric mean
Armillaria gallica FP102531 C6D	4.07 ± 1.61	10.21 ± 2.76	17.12 ± 1.87	8.93
Armillaria gallica EL8 A6F	3.20 ± 1.17	1.89 ± 1.26	15.42 ± 5.10	4.54
Fomes fomentarius TJV93 7 A3E	10.41 ± 1.81	21.26 ± 11.9	47.24 ± 28.68	21.87
Merulius tremellosus FP150849 C3F	13.52 ± 1.51	15.95 ± 1.79	43.91 ± 11.72	21.15
Phellinus hartigii DMR94 44 A10E	2.30 ± 0.27	12.86 ± 4.80	17.39 ± 12.86	8.01

Table 2: Hyphal extension rate measured for each isolate at 10, 16 and 22°C

Isolate	10 °C	16 °C	22 °C
Armillaria gallica FP102531 C6D	0.30 ± 0.05	0.36 ± 0.05	0.34 ± 0.06
Armillaria gallica EL8 A6F	0.18 ± 0.06	0.26 ± 0.05	0.38 ± 0.15
Fomes fomentarius TJV93 7 A3E	0.36 ± 0.08	1.28 ± 0.22	4.62 ± 0.24
Merulius tremellosus FP150849 C3F	3.40 ± 0.00	6.50 ± 2.24	8.33 ± 0.39
Phellinus hartigii DMR94 44 A10E	0.49 ± 0.12	1.26 ± 0.07	0.94 ± 0.10

Table 3: Some information about fungi

Isolate	Moisture Niche Width	Ranking	Moisture Tolerance	Decomposition Rate
Armillaria gallica FP102531 C6D	0.6069	0.2325	-0.3744	8.93
Armillaria gallica EL8 A6F	0.4194	0.1644	-0.2550	4.540
Fomes fomentarius TJV93 7 A3E	0.2399	0.2847	0.0448	21.870
Merulius tremellosus FP150849 C3F	0.2500	0.8383	0.5883	16.160
Phellinus hartigii DMR94 44 A10E	0.3165	0.4932	0.1767	7.100

According to the definition, the moisture tolerance means the difference between competitive ranking and moisture niche width. As long as the competitive ranking and moisture niche width are able to be measured by a value between [0,1], then we could easily calculate the difference between them, namely the moisture tolerance could be roughly quantified.

Based on the above analysis, firstly we need to get the data about the moisture niche width of

fungi, then use the moisture niche width of each fungus to divide the maximum value of this set of data, in this way can we measure the moisture niche width by a value between [0,1] [5].

Let us take fungus *Fomes fomentarius* TJV93 7 A3E as example, based on the data, the competitive ranking of this fungus is 0.28474739872575, moisture niche width is 1.19, q is 0.044828044. Then we donate that $T = 22$, $m = 0.007$, $n = 0.08$, $b = 0.25$. We assume that ground litter and woody fibers will not be replenished under laboratory conditions, so we donate that $r = 0$. According to the model, we can draw the function about x, y changing by time (t), refer to Figure 2:

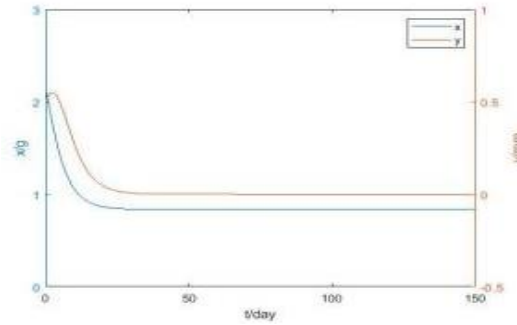


Figure 2: Changing image of fungi and decomposed substances

As we can see from Figure 2, when the woody fibers are sufficient at the beginning, the number of fungi has a slight growth. When the number of fungi reach to maximum value, the slope of woody fibers changing curve get to the largest, namely the decomposition rate is fastest. However, when woody fibers are inadequate, fungi began to die in large numbers, and the final value is 0. This is consistent with the growth of fungi at a time of limited resources. On the 122nd day, woody fibers' decomposition rate is 65.45%, the results we get is in accordance with the range of decomposition rate in Table 1. When it comes to a single fungus, Volterra-Logistic Model conform to the experimental result, so the model is reliable. [6]

3. Conclusions

If the wood fiber as fungus "food", then fungi as the predator, r_1 for wood fiber growth rate, y is the number of fungi, a wood fiber for fungal decomposition rate, considering the number of fungi will affect the decomposition of woody material. The number change of fungi is related to their death rate d , food mass x , the overall length of hyphae y and environmental capacity N . Considering different fungi competing, they will take up each other's resources, the ability of occupancy resources is σ . Finally, the relationship between growth rate a , mortality rate d and environmental temperature, humidity and moisture tolerance was established, and the final Lotka-Volterra-Logistic Model was obtained.

$$\begin{cases} \frac{dx}{dt} = x(r_1 - a_1 y_1 y_2) \\ \frac{dy_1}{dt} = [b_1 x - \frac{k_2(1 - q_1)}{2}] (1 - \frac{y_1}{N_1} - \sigma_1 \frac{y_2}{N_2}) y_1 \\ \frac{dy_2}{dt} = [b_2 x - \frac{k_2(1 - q_2)}{2}] (1 - \frac{y_2}{N_2} - \sigma_2 \frac{y_1}{N_1}) y_2 \end{cases} \quad (5)$$

According to the model established by us, the relationship between the temperature and the decomposition rate of wood fiber under laboratory conditions (The wood fiber itself does not grow.) is as follows Figure 3:

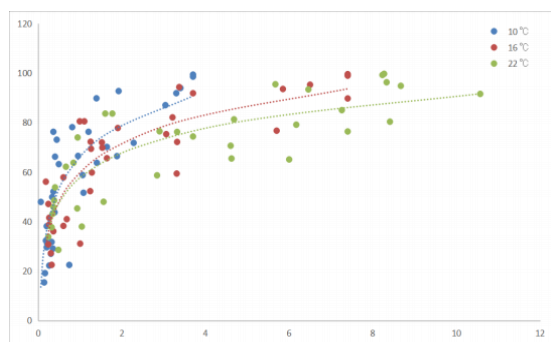


Figure 3: Relationship between temperature and decomposition rate of wood fiber

The relationship between the moisture resistance and the decomposition rate of wood fiber is shown in the Figure 4 as follows.

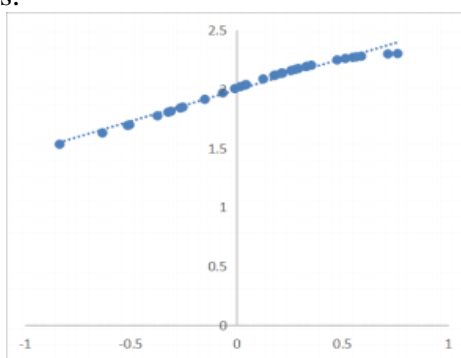


Figure 4: Moisture resistance and decomposition rate of wood fiber

In this paper, the decomposition rate of wood fiber and ground litter is analyzed by mathematical Model. On the basis of volterra model, we consider logistic Growth Model to improve it, so that the model is closer to the actual situation. However, this paper only analyzed the influence of temperature and humidity on fungi in the environment, ignoring other factors, resulting in a certain degree of deviation between the decomposition rate in our model and the reality. And the relationship between moisture resistance and mortality is not well considered, we simply assume that they are linearly correlated. To sum up, further optimization is needed.

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