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Cooperative Relaying System Combining OMA and NOMA for Cell-Edge and Cell-Center Users

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Abstract: In this letter, we propose a cooperative relaying system (CRS) combining orthogonal multiple access (OMA) and non-orthogonal multiple access (NOMA) for uplink transmission. In the proposed system, cell-center user (CCU) amplifies and forwards the cell-edge user (CEU) signal to base station (BS), meanwhile transmitting its own signals by NOMA. This win-win operation drives CCU and CEU to work together, and is particularly useful when CCU holds good channel conditions to BS but lacks radio resources. Both analytical and asymptotic expressions of the outage probability are derived for CEU and CCU. Simulation results demonstrate the large performance gains achieved over the benchmark schemes, owing to the adoption of the proposed system.

1. Introduction

Cooperative relaying systems (CRSs) have received tremendous attention to exploit spatial diversity [1]. Non-orthogonal multiple access (NOMA) is a promising multiple access principle to achieve superior spectral efficiency and has been extended to CRSs [2]. The authors in [2] proposed a NOMA base CRS where the users with better channel conditions are selected as relays for helping the others. Recently, the authors in [3-8] have investigated NOMA based CRSs for different transmission architectures, such as the uplink, the downlink and the composite (both uplink and downlink).

All of the above NOMA based CRSs only exploit NOMA and do not consider orthogonal multiple access (OMA) principle. However, OMA might be preferred for certain application scenarios rather than NOMA, such as a cell with a small number of users or the SNR is not large enough to support NOMA. To fulfill different requirements of varied services and applications, both OMA and NOMA will coexist in 5G networks and beyond. Also, these NOMA based CRSs assume that the relay is willing to offer help to other nodes without pursuing its own interest (e.g., delivering its own signals). But for the majority of wireless applications, users are selfish and prefer to transmit their own data prior to others.

The authors in [9-11] proposed the NOMA based CRSs for the downlink system, where OMA and NOMA are exploited. However, the schemes in [9-11] focused on the downlink system where the signal processing is completely different from the uplink system. Considering the uplink transmission is crucial for 5G networks and beyond [4], this letter proposes a novel NOMA based CRS for uplink to boost spectral efficiency. Since OMA and NOMA will coexist in future, an OMA-NOMA combined CRS is proposed, where a cell-edge user (CEU) seeks the help of a cell-center user (CCU) to communicate with base station (BS). Different from the existing NOMA based CRSs, the proposed system operates in a win-win way. Particularly, CCU helps forward the CEU information to BS, meanwhile transmitting its own information. Since CEU and CCU are simultaneously served, spectral efficiency can be effectively improved.

The authors in [1] show that for decode-and-forward (DF) and amplify-and-forward (AF) relaying protocols, diversity order is one and two, respectively. Thus, the AF relaying protocol is exploited by CCU to forward the CEU signal in the proposed system. Considering selection combining (SC) has the lower complexity in practice but achieves the same diversity order as maximum ratio combining (MRC) in the high-SNR regime, BS exploits SC to detect signals in this letter.

Notations—we denote the probability density function and the cumulative density functions of α by $f_\alpha(x)$ and $F_\alpha(x)$, respectively. $|\cdot|$ represents the Euclidean norm of a scalar.

2. System model

As shown in Fig. 1, we consider an uplink system consisting of a CEU, a CCU and a BS. To improve the reliability of the transmission from CEU to BS, CEU seeks helps from CCU by recruiting it as a relay. CEU and CCU operate in half-duplex mode. Channels experience independent and non-identically distributed Rayleigh fading that is constant during a time slot and changes from one slot to another independently. Particularly, the channel gains from CEU to CCU and BS are denoted by $g_{e,c}$ and $g_{e,b}$, and the gain from CCU to BS is denoted by $g_{c,b}$, with E[$|g_{e,c}|^2$]=$\lambda_{e,c}$, E[$|g_{e,b}|^2$]=$\lambda_{e,b}$, [$|g_{c,b}|^2$]=$\lambda_{c,b}$. The target data rate of BS is r, and $\gamma_{th}=2^r-1$ is the threshold for BS reliably decoding data. Without losing generality, it is assumed that the transmit powers are P_t at all transmitters, and the noise variances are σ^2 at all receivers.

We assume that CEU has its signal, denoted by x_e, to be transmitted to BS, and CCU has its k signals, denoted by x_c^i (i=1,2,…,k), to be transmitted to BS, with E[x_e]=E[x_c^i]=0 and E[$|x_e|^2$]=E[$|x_c^i|^2$]=1. The proposed system divides the CEU transmission time, which is dedicatedly allocated to CEU to transmit its signal, into two phases, namely OMA transmission phase and NOMA inspired forwarding phase.

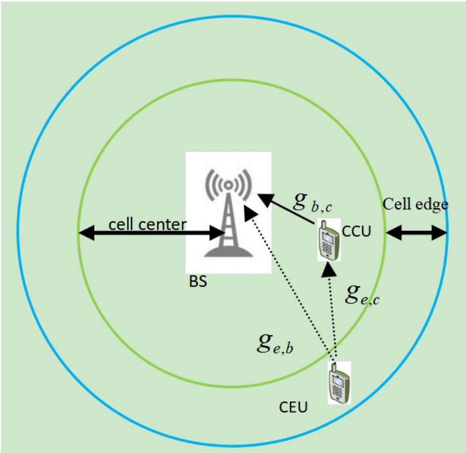

Figure 1: OMA-NOMA combined cooperative relaying system for uplink

2.1 OMA transmission phase

At this phase, CEU transmits it signal x_e using OMA. The received signal y_c^{I} at CCU, and y_b^{I} at BS are given by

$$y_c^{\mathrm{I}} = \sqrt{P_t} g_{e,c} x_e + \eta_c^{\mathrm{I}} \tag{1}$$

$$y_b^{\mathrm{I}} = \sqrt{P_t} g_{e,b} x_e + \eta_b^{\mathrm{I}} \tag{2}$$

where η_c^{I} and η_b^{I} are the background noise at CCU and BS, respectively. Therefore, the signal-to-noise ratio (SNR) of decoding signal x_e by BS at this phase is given by

$$\gamma_e^{\mathrm{I}} = \rho |g_{e,b}|^2 \tag{3}$$

where $\rho = P_t / \sigma^2$ denotes the transmit SNR at each transmitter.

2.2 NOMA inspired forwarding phase

CCU multiplies y_c^{I} by $G = \sqrt{w_0 P_t / (P_t | g_{e,c} |^2 + \sigma^2)}$, such that the transmit power for the superimposed CEU signal is $w_0 P_t$. CCU superimposes y_c^{I} and x_c^i (i=1,2,…,k) together, forming the superimposed signal X as follows

$$X = G y_c^{\mathrm{I}} + \sum_{i=1}^{k} \sqrt{w_i P_t} x_c^i \tag{4}$$

where w_i is the power allocation coefficient for signal x_c^i with $\sum_{i=1}^{k} w_i = 1 - w_0$. Since the main aim of CCU is to help CEU communicate with BS, CEU has the highest priority and the power allocation coefficient of the CEU signal x_e, i.e., w_0, is set to the highest value. For the k CCU signals, without loss of generality, we let $w_1 \geq w_2 \geq \ldots \geq w_k$, where w_i represents the power allocation coefficient of the CCU signal x_c^i (i=1,2,…,k). Thus, we have $w_0 \geq w_1 \geq \ldots \geq w_k$.

At NOMA inspired forwarding phase, CCU transmits signal X , and the received signal y_b^{II} at BS is given by

$$\begin{aligned} y_b^{\mathrm{II}} &= g_{c,b} X + \eta_b^{\mathrm{II}} \\ &= g_{c,b} \frac{\sqrt{w_0 P_t}}{\sqrt{P_t | g_{e,c} |^2 + \sigma^2}} \sqrt{P_t} g_{e,c} x_e + g_{c,b} \frac{\sqrt{w_0 P_t}}{\sqrt{P_t | g_{e,c} |^2 + \sigma^2}} \eta_c^{\mathrm{I}} + g_{c,b} \sum_{i=1}^{k} \sqrt{w_i P_t} x_c^i + \eta_b^{\mathrm{II}} \end{aligned} \tag{5}$$

where η_b^{II} is the background noise at BS at this phase.

Using y_b^{II}, BS first decodes signal x_e. After x_e is successfully decoded, BS removes x_e and then decodes signal x_c^i (i=1,2,…,k) by successive interference cancellation (SIC). Specifically, to detect x_c^i, BS will first detect x_c^j for $j<i$, and then successively removes the signal from its observation. Therefore, using y_b^{II}, the SNR of decoding x_e by BS is given by

$$\gamma_e^{\mathrm{II}} = \frac{w_0 \rho | g_{e,c} |^2 \rho | g_{c,b} |^2 / (\rho | g_{e,c} |^2 + 1)}{\sum_{j=1}^{k} w_j \rho | g_{c,b} |^2 + w_0 \rho | g_{c,b} |^2 / (\rho | g_{e,c} |^2 + 1) + 1} \tag{6}$$

The SNR of decoding x_c^i by BS is given by

$$\gamma_{c,i} = \frac{w_i \rho | g_{c,b} |^2}{\rho | g_{c,b} |^2 \sum_{j=i+1}^{k} w_j + 1}, i = 1,2,\ldots,k-1 \tag{7}$$

Particularly, the SNR of decoding x_c^k by BS is given by

$$\gamma_{c,k} = w_k \rho |g_{c,b}|^2 \tag{8}$$

3. Outage performance analysis

3.1 CEU

1) Outage Probability:

Let O_e denote the event that BS fails to decode CEU signal.

Theorem 1. *With the proposed system, the outage probability of CEU can be calculated as (9), where* θ *is defined in Appendix A* and $K_1(\,)$ *is the first-order modified Bessel function of the second kind.*

$$\Pr(O_e) = (1-\exp(-\frac{\gamma_{th}}{\lambda_{e,b}\rho})) \times (1-\exp(-\frac{\theta}{\lambda_{c,b}}-\frac{\theta}{\lambda_{e,c}})\frac{1}{\lambda_{e,c}}\sqrt{\frac{4\theta(\rho\theta+1)\lambda_{e,c}}{\rho\lambda_{c,b}}}K_1(\sqrt{\frac{4\theta(\rho\theta+1)}{\rho\lambda_{c,b}\lambda_{e,c}}})) \tag{9}$$

Proof. See Appendix A.

2) Asymptotic Analysis:

According to $K_1(x) \approx 1/x$ [12] and $\exp(x) \approx 1+x$ for a sufficiently high transmit SNR, the asymptotic expression of the CEU outage probability is given by

$$\Pr(O_e^{\infty}) = \frac{1}{\rho}\frac{\gamma_{th}}{\lambda_{e,b}}\left(\frac{\theta}{\lambda_{c,b}}+\frac{\theta}{\lambda_{e,c}}\right) = \frac{\gamma_{th}\theta}{\rho}\frac{1}{\lambda_{e,b}}\left(\frac{1}{\lambda_{c,b}}+\frac{1}{\lambda_{e,c}}\right) \tag{10}$$

where the superscript ∞ indicates the transmit SNR in the high-SNR regime.

We can see that $\Pr(O_e^{\infty})$ is proportional to $1/\lambda_{e,b}$ and $1/\lambda_{c,b}+1/\lambda_{e,c}$, corresponding to the direct path and the relaying path, respectively. Let d denote the diversity order of the CEU signal x_e. d can be calculated as

$$d = \lim_{\rho\to+\infty} -(\ln(\Pr(O_e^{\infty})/\ln(\rho)) = 2 \tag{11}$$

The above formula indicates that the proposed system achieves the full cooperative diversity of two for CEU signal.

3.2 CCU

1) Outage Probability:

Let O_c denote the event that BS fails to decode all of the CCU signals.

Theorem 2. With the proposed system, the outage probability of CCU can be calculated as (12), where $\phi_{\max}$ and ε are defined in Appendix B.

$$\begin{aligned}\Pr(O_c) = {} & \exp(-\frac{\gamma_{th}}{\lambda_{e,b}\rho})\left(1-\exp(-\frac{\phi_{\max}}{\lambda_{c,b}})\right)+(1-\exp(-\frac{\gamma_{th}}{\lambda_{e,b}\rho}))\times\left(1-\exp(-\frac{\theta}{\lambda_{e,c}})+(1-\exp(-\frac{\phi_{\max}}{\lambda_{c,b}}))\exp(-\frac{\varepsilon}{\lambda_{e,c}})+\frac{(\varepsilon-\theta)\pi}{2n+2}\sum_{i=0}^{n}(\sqrt{1-\left(\cos(\frac{(2i+1)\pi}{2n+2})\right)^2}\times\right. \\ & \left. F_{|g_{c,b}|^2}(\frac{\gamma_{th}(\rho(\frac{\varepsilon-\theta}{2}\cos(\frac{(2i+1)\pi}{2n+2})+\frac{\varepsilon+\theta}{2})+1)}{(\frac{\varepsilon-\theta}{2}\cos(\frac{(2i+1)\pi}{2n+2})+\frac{\varepsilon+\theta}{2})(w_0\rho^2-\gamma_{th}\rho^2\sum_{j=1}^{k}w_j)-\gamma_{th}\rho})\times f_{|g_{e,c}|^2}(\frac{\varepsilon-\theta}{2}\cos(\frac{(2i+1)\pi}{2n+2})+\frac{\varepsilon+\theta}{2}))\right)\end{aligned} \tag{12}$$

Proof. See Appendix B.

2) Asymptotic Analysis:

Exploiting the same rationale as (10), the asymptotic expression of the CCU outage probability can be obtained.

4. Numerical results

For a fair comparison with our system, we devise two benchmark systems in the following:

1) The OMA based relaying system (OMA-RS). It divides the CEU transmission time into two phases and considers time-division multiple access. After CEU transmits its signal in the first phase, a fraction of ξ of the second phase is used for CCU to relay CEU signal, and the remaining fraction of 1-ξ of the second phase is further divided into k time slots to dedicatedly transmit k signals of CCU.

2) The OMA based direct system (OMA-DS). CEU transmits its signal to BS using OMA during the CEU transmission time, without relaying.

Without loss of generality, we assumed ξ=0.5, $\lambda_{e,b}$=0.01 and $\lambda_{e,c}$=$\lambda_{c,b}$=0.05, with r being 0.3 bps/Hz at BS. The power allocation coefficients are set as w_0=0.7, w_1=0.2, and w_2=0.1. Fig. 2 shows the outage performance as a function of transmit SNR. As we can see, exploiting spatial diversity can effectively improve the reliability of the transmission from CEU to BS. The proposed system achieves much better outage performance for both CEU and CCU than OMA-RS. The reason behind is that CEU and CCU can be simultaneously served during the NOMA inspired forwarding phase in the proposed system. For example at SNR=30dB, the CEU outage probabilities of the proposed system and OMA-RS are 0.00205 and 0.00296 respectively, and the CEU outage performance gain is 30.7% by the proposed system over OMA-RS. The CCU outage probabilities of the proposed system and OMA-RS are 0.0981 and 0.165 respectively, and the CCU outage performance gain is 68.2% by the proposed system over OMA-RS.

We can increase ξ to improve CEU outage performance of OMA-RS, but increasing ξsignificantly decreases its CCU outage performance. For OMA-RS at SNR=30dB, we increase ξ to achieve the same CEU outage performance as the proposed system, but the CCU outage probability of OMA-RS increases to 0.325, which is 2.31 times larger than that of the proposed system.

It is worth pointing out that the analytical results perfectly match simulations and the asymptotic results are very tight in the high-SNR regime in Fig. 2.

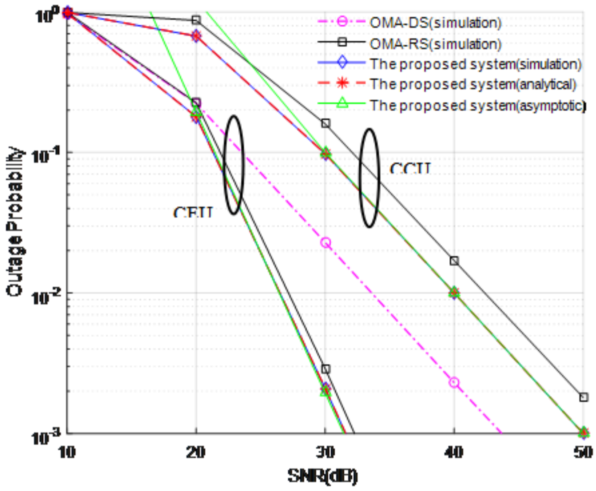

Figure 2: The outage probability

5. Conclusion

We propose an OMA-NOMA combined CRS to enhance the uplink performance, where CCU relays CEU signal to BS by exploiting AF protocol, meanwhile transmitting its own signals by employing NOMA. The CEU signal achieves the full cooperative diversity order of two in the proposed system. Also, the proposed system achieves a win–win situation for CEU and CCU and is particularly useful when CCU holds good channel conditions but lacks radio resources. The analytical and asymptotic expressions of the outage probability are derived for CEU and CCU. Simulations validate the analytical models' accuracy and demonstrate the proposed scheme's effectiveness.

APPENDIX A

Since BS exploits SC to detect signals, $\Pr(O_e)$ can be calculated as

$$\Pr(O_e) = (1-\Pr\{\gamma_e^{\mathrm{I}} > \gamma_{th}\})(1-\Pr\{\gamma_e^{\mathrm{II}} > \gamma_{th}\}) \tag{13}$$

where $\Pr\{\gamma_{\mathrm{e}}^{\mathrm{I}} > \gamma_{th}\} = 1 - F_{|g_{e,b}|^2}(\gamma_{th}/\rho) = \exp(-\gamma_{th}/(\lambda_{e,b}\rho))$.

For $\Pr\{\gamma_e^{II} > \gamma_{th}\}$, we have

$$\begin{aligned}\Pr\{\gamma_e^{\mathrm{II}} > \gamma_{th}\} &= \Pr\{\frac{w_0\rho|g_{e,c}|^2\ \rho|g_{c,b}|^2/(\rho|g_{e,c}|^2+1)}{\rho|g_{c,b}|^2\sum_{j=1}^{k}w_j + w_0\rho|g_{c,b}|^2/(\rho|g_{e,c}|^2+1)+1} > \gamma_{th}\}\\ &= \Pr\{|g_{c,b}|^2(|g_{e,c}|^2(w_0\rho^2-\gamma_{th}\rho^2\sum_{j=1}^{k}w_j)-\gamma_{th}\rho) > \gamma_{th}(\rho|g_{e,c}|^2+1)\}\end{aligned} \tag{14}$$

In the above formula, if $|g_{e,c}|^2(w_0\rho^2-\gamma_{th}\rho^2\sum_{j=1}^{k}w_j)-\gamma_{th}\rho$ <0, $\Pr\{|g_{c,b}|^2(|g_{e,c}|^2(w_0\rho^2-\gamma_{th}\rho^2\sum_{j=1}^{k}w_j)-\gamma_{th}\rho) > \gamma_{th}(\rho|g_{e,c}|^2+1)\}$ =0. Thus, we have

$$\begin{aligned}&\Pr\{\gamma_e^{\mathrm{II}} > \gamma_{th}\} = \Pr\{|g_{e,c}|^2(w_0\rho^2-\gamma_{th}\rho^2\sum_{j=1}^{k}w_j)-\gamma_{th}\rho > 0,\\ &|g_{c,b}|^2(|g_{e,c}|^2(w_0\rho^2-\gamma_{th}\rho^2\sum_{j=1}^{k}w_j)-\gamma_{th}\rho) > \gamma_{th}(\rho|g_{e,c}|^2+1)\}\\ &= \Pr\{|g_{e,c}|^2 > \frac{\gamma_{th}}{w_0\rho-\gamma_{th}\rho\sum_{j=1}^{k}w_j} = \theta, |g_{c,b}|^2 > \frac{\theta(\rho|g_{e,c}|^2+1)}{\rho(|g_{e,c}|^2-\theta)} = \phi_e\} = \int_\theta^\infty (1-F_{|g_{c,b}|^2}(\frac{\theta(\rho x+1)}{\rho(x-\theta)}))f_{|g_{e,c}|^2}(x)dx\\ &= \exp(-\frac{\theta}{\lambda_{c,b}}-\frac{\theta}{\lambda_{e,c}})\frac{1}{\lambda_{e,c}}\int_0^\infty \exp(-\frac{\theta(\rho\theta+1)}{\rho\lambda_{c,b}}\frac{1}{t})\exp(-\frac{1}{\lambda_{e,c}}t)dt = \exp(-\frac{\theta}{\lambda_{c,b}}-\frac{\theta}{\lambda_{e,c}})\frac{1}{\lambda_{e,c}}\sqrt{\frac{4\theta(\rho\theta+1)\lambda_{e,c}}{\rho\lambda_{c,b}}}K_1(\sqrt{\frac{4\theta(\rho\theta+1)}{\rho\lambda_{c,b}\lambda_{e,c}}})\end{aligned} \tag{15}$$

where the last step is according to eq. (3.324.1) in [13]. Combining (13) and (15), Theorem 1 can be proved.

APPENDIX B

The outage probability of CCU can be calculated as

$$\Pr(O_c) = \Pr(O_c\,|\,\gamma_e^{\mathrm{I}} > \gamma_{th})\Pr(\gamma_e^{\mathrm{I}} > \gamma_{th}) + \Pr(O_c\,|\,\gamma_e^{\mathrm{I}} < \gamma_{th})(1-\Pr(\gamma_e^{\mathrm{I}} > \gamma_{th})) \tag{16}$$

If $\gamma_e^{\mathrm{I}} > \gamma_{th}$, BS can use signal y_b^{I} to decode signal x_e. After successfully decoding x_e, $\{\gamma_{\mathrm{c},i} > \gamma_{th}\} \triangleq \{|g_{c,b}|^2 > \frac{\gamma_{th}}{w_i\rho-\sum_{j=i+1}^{k}w_j\rho} = \phi_i\}$ must be satisfied for BS reliably decoding signal x_c^i (i=1,2,...,k). Giving $\phi_{\max} = \max\{\phi_1,\phi_2,...,\phi_k\}$, $\{|g_{c,b}|^2 > \phi_{\max}\}$ must be satisfied for BS reliably decoding all k signals of CCU. $\Pr(O_c|\gamma_e^{\mathrm{I}} > \gamma_{th})$ can be calculated as

$$\Pr(O_c|\gamma_e^I > \gamma_{th}) = 1-\Pr\{|g_{c,b}|^2 > \phi_{\max}\} = 1-\exp(-\frac{\phi_{\max}}{\lambda_{c,b}}) \tag{17}$$

If $\gamma_{e}^{\mathrm{I}} < \gamma_{\mathrm{th}}$, BS cannot use signal y_b^{I} to decode signal x_e, and tries to exploit signal y_b^{II} to decode x_e. According to (15), using y_b^{II}, $\{|g_{e,c}|^2 > \theta\}$ and $\{|g_{c,b}|^2 > \phi_e\}$ must be satisfied for BS reliably decoding x_e. After successfully decoding x_e, $\{|g_{c,b}|^2 > \phi_{\max}\}$ must be satisfied for BS reliably decoding all k signals of CCU. Thus, $|g_{c,b}|^2 > \max(\phi_e, \phi_{\max})$ must be satisfied for BS to decode all CCU signals. Since $\{\phi_e > \phi_{\max}\} \triangleq$ and ε is always larger than θ, we have $\{|g_{e,c}|^2 < \dfrac{\gamma_{th}\phi_{\max} + \gamma_{th}}{\phi_{\max}(w_0\rho - \gamma_{th}\rho\sum_{j=1}^{k} w_j) - \gamma_{th}\rho} = \varepsilon\}$

$$\Pr(O_c \mid \gamma_e^{\mathrm{I}} < \gamma_{th}) = 1 - \Pr\{|g_{e,c}|^2 > \theta, |g_{c,b}|^2 > \max(\phi_e, \phi_{\max})\}$$

$$= 1 - \int_\theta^\infty (1 - F_{|g_{c,b}|^2}(\max(\phi_e,\phi_{\max}))) f_{|g_{e,c}|^2}(x)dx = \underbrace{\int_0^\theta f_{|g_{e,c}|^2}(x)dx}_{I_1} + \underbrace{\int_\theta^\varepsilon F_{|g_{c,b}|^2}(\phi_e) f_{|g_{e,c}|^2}(x)dx}_{I_2} + \underbrace{\int_\varepsilon^{+\infty} F_{|g_{c,b}|^2}(\phi_{\max}) f_{|g_{e,c}|^2}(x)dx}_{I_3} \tag{18}$$

where $I_1 = 1 - \exp(-\theta/\lambda_{e,c})$, $I_3 = (1 - \exp(-\phi_{\max}/\lambda_{c,b}))\exp(-\varepsilon/\lambda_{e,c})$ and I_2 can be calculated as

$$I_2 = \int_\theta^\varepsilon F_{|g_{c,b}|^2}\left(\frac{\gamma_{th}(\rho x + 1)}{x(w_0\rho^2 - \gamma_{th}\rho^2\sum_{j=1}^{k} w_j) - \gamma_{th}\rho}\right) f_{|g_{e,c}|^2}(x)dx \tag{19}$$

$$= \frac{\varepsilon-\theta}{2}\int_{-1}^{1} F_{|g_{c,b}|^2}\left(\frac{\gamma_{th}(\rho(\frac{\varepsilon-\theta}{2}t + \frac{\varepsilon+\theta}{2}) + 1)}{(\frac{\varepsilon-\theta}{2}t + \frac{\varepsilon+\theta}{2})(w_0\rho^2 - \gamma_{th}\rho^2\sum_{j=1}^{k} w_j) - \gamma_{th}\rho}\right) \times f_{|g_{e,c}|^2}\left(\frac{\varepsilon-\theta}{2}t + \frac{\varepsilon+\theta}{2}\right)dt$$

$$\approx \frac{(\varepsilon-\theta)\pi}{2n+2}\sum_{i=0}^{n}\left(\sqrt{1-\left(\cos(\frac{(2i+1)\pi}{2n+2})\right)^2} \times F_{|g_{c,b}|^2}\left(\frac{\gamma_{th}(\rho(\frac{\varepsilon-\theta}{2}\cos(\frac{(2i+1)\pi}{2n+2}) + \frac{\varepsilon+\theta}{2}) + 1)}{(\frac{\varepsilon-\theta}{2}\cos(\frac{(2i+1)\pi}{2n+2}) + \frac{\varepsilon+\theta}{2})(w_0\rho^2 - \gamma_{th}\rho^2\sum_{j=1}^{k} w_j) - \gamma_{th}\rho}\right) \times f_{|g_{e,c}|^2}\left(\frac{\varepsilon-\theta}{2}\cos(\frac{(2i+1)\pi}{2n+2}) + \frac{\varepsilon+\theta}{2}\right)\right)$$

where the last step exploits Gaussian–Chebyshev quadrature [12] to find an approximation for the above formula.

Combining (16), (17), (18) and $\Pr\{\gamma_e^I > \gamma_{th}\} = \exp(-\gamma_{th}/(\lambda_{e,b}\rho))$, Theorem 2 can be proved.

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