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Construction of the Evaluation Framework of Senior High School Students' Key Abilities in Mathematical Modeling from the Situational Perspective

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Abstract: The evaluation of students' mathematical modeling literacy has attracted more and more attention from all walks of life in the international community. Because of the characteristics of mathematical modeling that connects the real world with the mathematical world, under the perspective of classification of situations, the ability of model hypothesis, modeling solution, and testing and promotion are taken as the dimensions of evaluating students' key abilities in mathematical modeling, thus students' key abilities in modeling are divided into three levels. The purpose is to provide reference for the evaluation of students' mathematical modeling literacy from the perspective of context.

1. Research background

With the continuous development of the current society, more and more educators pay attention to the core quality of mathematics. The Opinions on Comprehensively Deepening the Curriculum Reform and Implementing the Fundamental Task of Building Virtue and Cultivating People issued by the Ministry of Education defines the connotation of the core quality as: a necessary character and key ability that students should have that can meet the needs of lifelong development and social development [1]. The six core mathematical qualities proposed in the Curriculum Standards for Ordinary High Schools (2017 Edition) (hereinafter referred to as the "curriculum standards") include mathematical abstraction, logical reasoning, mathematical modeling, intuitive imagination, mathematical operations and data analysis [2]. Mathematical key competence is an essential component of mathematical core competence, which ensures the unity of knowledge and ability in the connotation of mathematical core competence [3]. As the most obvious part that can highlight mathematical modeling literacy, the key ability of mathematical modeling has gradually become the focus of current education research. At the same time, the cultivation of the key ability of mathematics can better highlight the ability orientation of "people-oriented" that the goal of mathematical education has changed from mainly teaching basic mathematical knowledge and basic skills to adapting to the development of the times. Combined with mathematical modeling itself, it is a comprehensive and creative activity. In the modeling process, students need to analyze the situation in real life, find problems and put forward problems on this basis, then solve the problem through the knowledge learned, and finally return to the real situation to solve the real problem. This requires students not only to stay at the level of knowledge, but also to put forward higher requirements for students' comprehensive ability. Figuratively, mathematical modeling allows students to experience the whole process of observing the real world with mathematical "eyes", describing the real world with mathematical "language", and exploring the real world with mathematical "thinking".

The exploration of students' mathematical modeling literacy cannot be separated from the research on the key abilities of mathematical modeling. To some extent, the key abilities of mathematical modeling externalize students' mathematical modeling literacy. In fact, the exploration of key abilities of mathematical modeling lays a foundation for the research on mathematical modeling literacy, and also provides a theoretical basis for the development of literacy. Therefore, the research on mathematical modeling, In particular, the evaluation research is more instructive for the improvement of students' mathematical core literacy, and combines the classification perspective of the situation to explore students' key abilities in mathematical modeling, so that the key abilities in mathematical modeling can be better reflected in the process.

2. Mathematical modeling and situation

When the core literacy has been widely concerned by the education sector and has become the main direction of the new round of education curriculum reform in China, more and more education researchers have begun to pay attention to the relationship between the mathematical core literacy and the situation. Therefore, as one of the six core mathematical literacy, the relationship between mathematical modeling and situation has both commonalities and characteristics with the rest of the relationship between mathematical core literacy and situation.

In pedagogy, the word "situation" can be used as the environment and background for students to participate in learning activities, providing students with space to think about problems, so that students can put forward problems and solve problems in this environment. On the other hand, mathematical context is the precondition, basis and background for forming mathematical concepts, finding mathematical problems, proposing and analyzing mathematical problems and finally solving mathematical problems [4]. In the Curriculum Standards, it is mentioned that the mathematical situation can be roughly divided into three types: real situation, pure mathematical situation and scientific situation. The real situation is composed of the problems that students face in real life, which are closely related to the students' life, such as family, society and the future occupation that they will encounter; Pure mathematical situation is mainly composed of some pure mathematical problems after mathematical abstraction, such as abstract concepts and abstract figures involved in mathematics; Compared with the first two situations, the scientific situation is far away from students' life. It is a problem situation that ordinary students cannot fully grasp and understand. It mainly refers to the problem situations that students may encounter in scientific experiments and scientific movement. According to the students' familiarity with situations, each situation can be subdivided into three levels: familiarity, relevance and synthesis. When a student solves the corresponding problems in the context of different levels, it can be considered that he has reached the corresponding ability level in mathematical modeling.

A person in a specific situation will have the appropriateness and attitude to meet the needs of the situation when he has certain qualities [5]. Scholars of the Organization for Economic Cooperation and Development (OECD) believe that literacy is a kind of ability to use knowledge, skills and attitudes to meet the key requirements in specific situations. Specifically, promoting individuals to meet their most important needs in various situations is one of the necessary conditions for the formation of mathematical core literacy, and individual mathematical core literacy plays a synergistic role in complex situations [6]. PISA assessment project specifically assesses students' performance in solving problems in real situations according to different real life backgrounds, so as to reflect their mathematical literacy. In addition, the importance of "real situation" of mathematical modeling is also emphasized in PISA2021 mathematical literacy evaluation framework, and mathematical modeling highlights the important role of mathematical knowledge and ability in solving real problems [7]. Mathematical modeling does not start from the real situation to solve mathematical problems. What is more important is to use mathematical knowledge and skills to retrace the real situation, and use mathematical knowledge and ability to help us understand the real world more easily. The "situation" in PISA evaluation project mainly refers to: personal situation, social situation and occupational situation. Chang Lei, a domestic scholar, believes that to promote the development of core literacy is to enable students to think in real problem situations and experience the whole process of solving practical problems in person, that is, to combine knowledge learning with diversified real situations [6].

Professor Shi Ningzhong pointed out that the basic ideas of mathematics include abstraction, reasoning and model, and the "model" idea directly reflects the extensive application value of mathematics [8]. Mathematical modeling represents the "model" idea among them. Mathematical modeling is to use the "vision of mathematical models" to abstract the actual problems mathematically, express the real situation with "mathematical language", and think about the problems in the real world with "model ideas", which reflects the extensive mathematical knowledge in life. Since mathematical modeling itself is closely related to mathematical situations, if mathematical modeling is the "bridge" between real life and the mathematical world, then mathematical situations are the "cornerstone" of building a "bridge", so that students can experience the close connection between the mathematical world and the real world in real situations. It can be seen that, from the field of natural science to the field of social science, or from the scope of professional research to daily life and production, mathematical modeling depends on rich and colorful real situations. Through the process of thinking in real situations to analyze problems, and finally solve problems, students' mathematical modeling literacy gradually permeates [6].

The cultivation of mathematical modeling literacy cannot be separated from the problem solving ability of students in complex situations. In a simple and familiar situation, students can often quickly associate the learned mathematical model to solve the actual problem. However, in a complex and unfamiliar situation, students' poor understanding of the situation will lead to problems in problem solving, so the model established cannot completely solve the problems in the real situation. It can be seen that, for students, the degree of familiarity with real situations is different, so the requirements for students' mathematical modeling ability will be different, that is, the mathematical modeling ability required to solve mathematical problems in familiar situations is low, while in unfamiliar real situations, students' mathematical modeling ability will be required to be high, so the evaluation of mathematical modeling literacy should be combined with rich situations.

3. Connotation of mathematical modeling literacy

The scientific model formed by mathematical logic method and mathematical language is called mathematical model. It uses mathematical structure expressions formed by mathematical methods such as generalization, abstraction and operation to describe the change law of practical problems and the characteristics of research objects [9]. Mathematical modeling is to simplify the actual problems in the real situation through thinking, and express the real problems in mathematical language through analysis, so as to establish and solve the mathematical model through mathematical knowledge, and finally return to the real situation to verify that the rationality of the model has solved the real problems. This process can generally be expressed as: finding problems, analyzing problems, establishing models, calculating solutions, explaining results, modifying models come to conclusion. The mathematical modeling process can be represented as Figure 1 below.

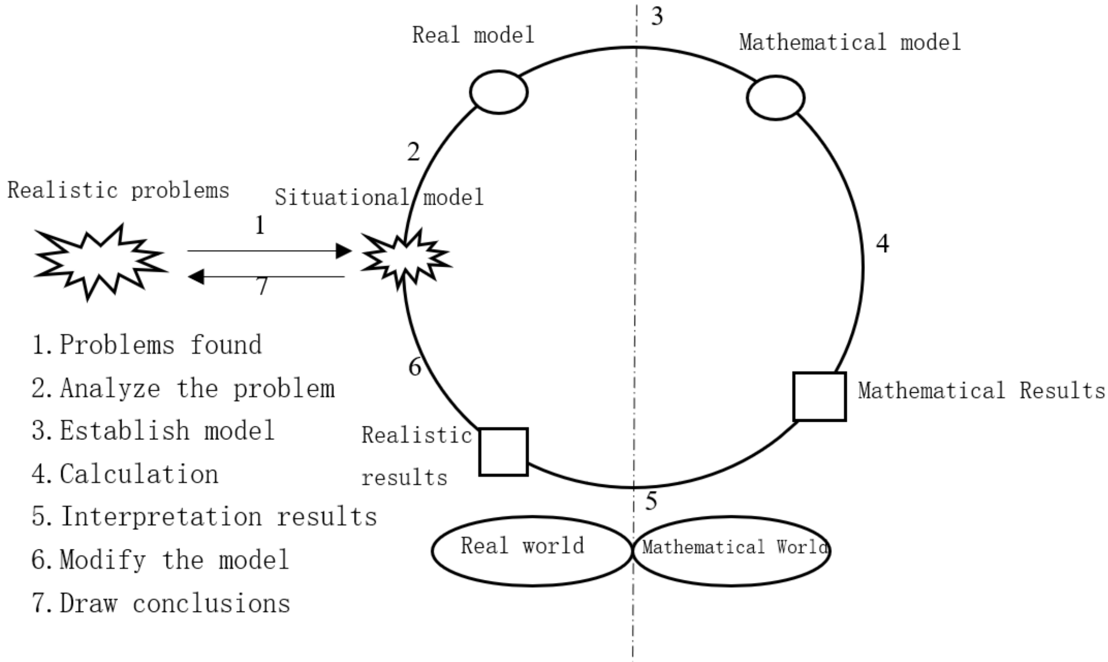

Figure 1: Structure Diagram of Mathematical Modeling Process

Understand mathematical modeling from the perspective of situation. Mathematical modeling is to extract the situation model from the real situation, transform the situation model into a real model through understanding the situation and simplifying the problem, and then transform the real model into a mathematical model through mathematical knowledge and broader knowledge for solution. Mathematical modeling connects the real world and the mathematical world in the whole process, and the situation here serves as the premise guarantee for the normal development of mathematical modeling activities.

In the Curriculum Standards, it is described as follows: "To improve students' mathematical literacy, it is necessary to guide students to learn to observe the world with mathematical eyes, learn to think about the world with mathematical thinking, and learn to express the world with mathematical language [2]. Mathematical modeling corresponds to the" expressing the world with mathematical language "in the" three conferences ". Mathematical modeling is to use mathematical language to express mathematical problems in life with a unified model. The vision, language and thinking of mathematics all correspond to the behavior of individuals in reality. In the process of mathematical modeling, it can also be well understood that the ability to analyze practical problems with mathematical knowledge, explore problems in real situations with mathematical thinking, and describe problems in the real world with mathematical language is an interpretation of the whole process of mathematical modeling. In order to enable students to gradually achieve the "three skills" in the process of mathematical modeling, students should be placed in the real situation as much as possible in the training of mathematical modeling. The practical problems closely related to students' lives can more cause students to observe, think and express problems.

According to the Curriculum Standards, students can improve their ability to find and raise problems, analyze and solve problems from the perspective of mathematics, which is the "four abilities" that students need to obtain [2]. This perspective is based on the process of finding, posing, analyzing and solving problems, which is one-to-one with the whole process of finding, posing, analyzing and finally solving problems in the real situation of mathematical modeling. The teaching method of taking problems as inspiration, taking activities as practice carrier, and using problems throughout the whole teaching process is conducive to the cultivation of mathematical core literacy [14]. Similarly, in the teaching of mathematical modeling, through the introduction of problem situations, students' thinking can be inspired in the whole teaching process, so that students' mathematical modeling literacy can be gradually improved in this process [10].

Based on the above points of view, the author combines the perspective of situation analysis, integrates the views of "three meetings" and "four abilities", and understands the connotation of the key ability of mathematical modeling as: the key ability of mathematical modeling is embodied in that students find and put forward problems in different situations, use mathematical knowledge to analyze problems according to different situations, and establish appropriate mathematical models; Using mathematical knowledge and methods to solve the model can test the model in real situations, improve and perfect the model, and finally return to the real world to solve problems in real situations. Through the cultivation of students' key abilities in mathematical modeling, they can observe the real world from the perspective of models, think about the real world with the thinking of models, and explain the real world with mathematical representation language.

4. Construction of evaluation framework for key capabilities of mathematical modeling

The evaluation of the key ability of mathematical modeling is the comprehensive evaluation of the students' performance at each stage and the mathematical language, mathematical thinking and mathematical methods they use to build models. In recent years, many researchers have studied the evaluation of students' mathematical modeling literacy. In the Curriculum Standards, the evaluation grade of mathematical modeling literacy is divided into three levels, reflecting the four aspects of mathematical core literacy: "situations and problems", "knowledge and skills", "thinking and expression", "communication and reflection", which is one of the dimensions of academic quality evaluation [2]. This evaluation framework is the most widely spread. It is also an evaluation framework that most teachers are familiar with, but the definition and differentiation of "familiar situation", "related situation" and "comprehensive situation" in this framework is not enough, and the statement is too vague. How to be "familiar", "related" and "comprehensive", so it lacks operability for actual teaching. Taking several versions of senior high school mathematics textbooks as the research object, Professor Li Baozhen made a comprehensive comparison of the mathematical modeling problem situation of the three versions of textbooks from four aspects: "content distribution and arrangement", "type and quantity", "authenticity level", and "characterization characteristics" [11]. In the dimension of "level of authenticity", mathematical modeling problem situations are divided into constructive situations, quasi real situations, real situations and pure real situations according to the degree of authenticity of the known conditions constructed by mathematical modeling problems. For teachers, the individual differences of students' understanding of problems and familiarity with situations are reduced to a certain extent. In the evaluation, it is only necessary to divide the levels according to the level of authenticity of the problem situations. For students, the level of the situation in which they work on the same problem is the same. Combined with the three levels of situation dimension division in the Curriculum Standards, the situation types of this evaluation framework are divided into constructive situation, authentic situation and pure reality situation.

For the evaluation of the key ability of mathematical modeling, it is mainly to recognize the way to evaluate students, and which behavior or measurable indicators of students can affect their mathematical modeling ability. The high school mathematics curriculum standard defines the core quality of mathematics as a comprehensive ability, while Ma Yunpeng's views in his articles are similar [12], focusing on the investigation of "mathematical ability". This ability based approach shows the quality of thinking that the quality is internalized in the mind and externalized, so that teachers can better evaluate the students' core quality of mathematics in a measurable form. In terms of the evaluation of mathematical modeling literacy, many scholars also evaluate the mathematical modeling ability. For example, Blum, a foreign scholar, divides the entire mathematical modeling process into seven sub processes: understanding problems - simplifying or structuring - mathematizing - mathematical operation - interpretation - verification - expression [13]. He called the mathematical modeling ability shown in the process of mathematical modeling as the mathematical modeling sub ability, and then used the sub ability as the evaluation dimension to evaluate the students' mathematical modeling ability, thus promoting the improvement of students' mathematical modeling literacy. Mathematical modeling ability is a comprehensive mathematical ability, rather than a single mathematical ability, and it contains some corresponding mathematical modeling sub abilities, which are shown through each stage of mathematical modeling. When students are engaged in mathematical modeling activities, their measurable ability can be used as a dimension to evaluate their mathematical modeling literacy. The following is a detailed description of the evaluation dimensions and levels.

4.1. Evaluation dimensions of key capabilities in mathematical modeling

From the point of view of mathematical modeling stage, it can be divided into model hypothesis stage, modeling solution stage and debugging promotion stage. At each stage, students can also use the key abilities reflected in this stage as the indicators for investigation. Teachers can judge the level of mathematical modeling ability achieved by students through their different ability levels in different modeling stages. In the model hypothesis stage, students discover problems from the mathematical perspective, analyze and simplify problems with model thinking, and then make assumptions about variables, so as to achieve the transformation from the real world to the mathematical world. In this stage, students' model hypothesis ability is highlighted; In the modeling and model solving stage, students can achieve a higher ability than the previous stage. They can express problems mathematically through mathematical language on the basis of the previous stage, build mathematical models, and use the knowledge learned to solve mathematical models to get results. In this stage, students' modeling and model solving ability is mainly investigated; In the third debugging and promotion stage, students debug the model, verify the rationality of the mathematical conclusion in the real situation, and explain its popularization. In this stage, students' ability to test and promote is mainly investigated.

Therefore, taking the model assumption ability, modeling and interpretation ability and testing and promotion ability as the three dimensions of the evaluation framework is the refinement of each stage of mathematical modeling, and is also the requirement for students' ability to progress step by step.

4.2. Level division of key mathematical modeling capabilities

In the Course Standard, the mathematical modeling literacy is divided into Level 1, Level 2 and Level 3, which are progressive from low to high. The main basis for the division of the evaluation framework is that the students' mathematical modeling ability in different situations is gradually enhanced under the "situation" constructive situation, authenticity situation and pure reality situation with different levels of authenticity. From the perspective of situation, evaluating students' mathematical modeling literacy can better reflect the characteristics of mathematical modeling. Therefore, the author continues this three level division method, will emphasize the level of each situation, and the specific operational definition at each level, will better combine mathematical modeling with the situation, and give play to the characteristics that mathematical modeling can connect the real situation and the mathematical world.

4.3. Construction of evaluation framework

To sum up, from the perspective of the situation, the key abilities of senior high school students in mathematical modeling are evaluated. The levels of key abilities in mathematical modeling are divided into three levels, and specific operational definitions are given, as shown in Table 1 below.

Table 1: Evaluation framework for key capabilities of mathematical modeling

Level	Model assumption capability	Modeling and solving capability	Inspection and promotion ability
Level 1	In the constructive situation, students can simply understand the conditions in the situation and make preliminary model assumptions.	In the constructive situation, the learned model can be preliminarily identified, and the model can be simply solved.	In the constructive context, the significance of the results of the model in the real situation is reasonably explained.
Level 2	In a real situation, you can transfer a simple model to a more complex situation, make model assumptions, and make reasonable explanations.	In a realistic situation, the relationship between models can be understood, the learned model can be transferred to a new situation, and the model can be solved by using mathematical tools.	In the authentic situation, the model can be extended to other situations.
Level 3	In a pure real situation, you can flexibly use the knowledge you have learned to analyze complex situations, and then make model assumptions, and can reasonably explain them.	In a purely realistic situation, we use interdisciplinary knowledge and mathematical thinking to build a model, and use comprehensive technology to solve this model.	In a purely realistic situation, we can guess the development of the mathematical model and have our own reasonable explanation.

5. Conclusion

In the future, the research on mathematical modeling will not only stay at the level of ability, but the core quality of mathematical modeling includes mathematical modeling ability and necessary character and values. However, the evaluation of the key ability of mathematical modeling is the basis of our research on the core quality of mathematical modeling. Therefore, the core quality of mathematical modeling reflects and the establishment of the evaluation framework at this stage is only the first step. After that, it is really necessary to use this evaluation framework to do empirical research, and building a reasonable evaluation framework for the key abilities of mathematical modeling from the context perspective is an important guarantee for the reliability and validity of empirical research. In the future, it will be my next step to continue to explore the evaluation dimension of mathematical modeling core literacy and empirical investigation from the situational perspective.

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