

Multi-objective optimization method based on regarding the high power research

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Abstract: The development of science and technology is often accompanied by higher requirements for energy. Wave energy is a kind of renewable energy which can not be ignored. It is widely distributed in the world and has a wide utilization prospect. In this paper, the motion of a "float" wave energy device is analyzed, and it is decomposed into a one-dimensional case and a two-dimensional case for analysis. By adjusting the relevant parameters in the wave energy device, the efficiency of energy conversion is optimized, and the optimal energy conversion is obtained in the case of one and two dimensions. The research has reference value for the manufacture and production of this type of wave energy device.

1. Introduction

With the development of science and technology year by year, human development is faced with the problem of unbalance between supply and demand and the risk of environmental pollution [1], so it is necessary for people to constantly explore ways to collect and utilize renewable energy. The energy conversion efficiency of the wave energy device we are going to study is one of the key issues in the large-scale utilization of renewable wave energy [2].

2. The construction of one-dimensional model

2.1 Conventions of coordinate systems

In the one-dimensional case, if we only consider the vertical motion [3], we only have one degree of freedom in the z direction. Its effect is shown in Figure 1:

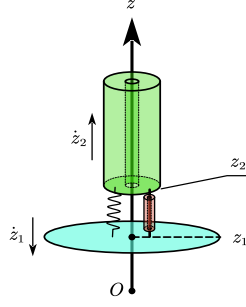


Figure 1: The coordinate system under one-dimensional simplification

O represents the initial position of the compartment (the bottom of the spring) [4], as the origin of coordinates. Figure 1 shows that at some point, the base and oscillator are above the origin of coordinates, but the motion direction is opposite. In order to simplify the calculation, we take the float as the reference frame as shown in Figure 2, so that the unknown parameters are the floating motion displacement $z_1(t)$ and relative displacement $z_r(t)$.

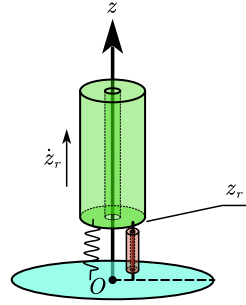


Figure 2: The float is taken as the reference frame diagram

2.2 The establishment of equation of motion in one-dimensional case

First, it is assumed that in the one-dimensional simple case, the vibration amplitude is not so large that the barrier layer of the wave energy device rises above sea level, or the device is completely submerged in water. This paper will verify this assumption by calculation. It is calculated that the system will not be completely submerged when the maximum amplitude of the float does not exceed 1.0000m.

Newton's second law is given in absolute frame of reference for the whole system:

$$(m_1 + m_a)\ddot{z}_1 + m_2(\ddot{z}_1 + \ddot{z}_r) = f \cos \omega t - \rho g \Delta V - \beta_{ez} \dot{z}_1 \quad (1)$$

In the above equation, m_a represents the added mass and ΔV represents the change in displacement. Under the above assumptions, $\Delta V = \pi R^2 z_1$. The right side of the equal sign represents the wave excitation force respectively; Hydrostatic resilience; Wave damping force. Then write the equation of motion of the oscillator in the floating coordinate system:

$$m_2 \ddot{z}_r = -k_z z_r - \beta_z \dot{z}_r - m_2 \ddot{z}_1 \quad (2)$$

In the above equation, the right side of the equal sign respectively represents the spring tension, the damping force of the damper and the inertial force of the non-inertial reference frame. And the reason I don't care about the force of gravity here is because in the initial case the force of gravity has been balanced out, so I just care about the change in the force of spring.

2.3 Float moment of inertia model

Consider the moment of inertia J_1 of the float about its axis. J_1 can be divided into the moment of inertia of cylinder shell and cone shell J_1^* and J_2^* . The moment of inertia of the cylinder can be obtained by the parallel axis theorem. This paper obtained:

$$J_1^* = \frac{m_1^*}{12} (6R^2 + H_1^2) + \frac{m_1^*}{4} H_1^2 = m_1^* \left(\frac{1}{2} R^2 + \frac{1}{3} H_1^2 \right) = 14035.3 \text{ kg} \cdot \text{m}^2 \quad (3)$$

The shell of the cone is divided into thin bands of height, as shown in Figure. 3:

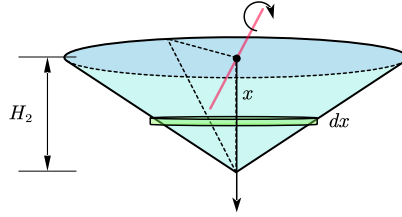


Figure 3: Microdivision of pyramidal shell

Each ring band can be viewed as a ring. It is known that the moment of inertia of a circle with mass μ and radius a around the diameter is $\frac{1}{2} \mu a^2$. According to this formula [5], the parallel axis theorem is used for integration, and the following can be obtained:

$$J_2^* = \int \frac{R^2}{2} \left(1 - \frac{x}{H_2} \right)^2 dm + x^2 dm \quad (4)$$

The former term in the above equation represents the moment of inertia of each ring band about its diameter, and the latter term is the parallel axis theorem. Then, write the relationship between the micro-element mass dm and the investigation height h :

$$dm = m_2^* \cdot \frac{2\pi R \cdot \frac{H_2 - x}{H_2}}{\pi R \sqrt{R^2 + H_2^2}} \cdot \frac{\sqrt{R^2 + H_2^2} dx}{H_2} = m_2^* \cdot \frac{2(H_2 - x)}{H_2^2} dx \quad (5)$$

The final calculation is:

$$J_1 = J_1^* + J_2^* = 14340.6 \text{ kg} \cdot \text{m}^2 \quad (6)$$

2.4 Calculation of the height of the center of gravity of the float

The axis of rotation is taken as the origin of coordinates, and the positive direction of Z-axis is built outward along the central axis. The coordinate L_c of the center of gravity can be expressed as:

$$L_c = \frac{\int_{z=-H_2}^{z=H_1} z dm}{m_1} \quad (7)$$

According to the geometric relation and quoting equation (5), the above equation can be rewritten as:

$$L_c = \frac{\int_0^{H_1} \frac{m_1^*}{H_1} z dz - \int_0^{H_2} m_2^* \frac{2(H_2 - z)}{H_2^2} z dz}{m_1} \quad (8)$$

Substitute in the data and obtain:

$$L_c = 1.189m \quad (9)$$

2.5 The damping coefficient of linear damper is constant

To solve the one-dimensional case, we have the following two constraint equations:

$$\begin{cases} (m_1 + m_a)\ddot{z}_1 + m_2(\ddot{z}_1 + \ddot{z}_r) = f \cos \omega t - \rho g \Delta V - \beta_{ez} \dot{z}_1 \\ m_2 \ddot{z}_r = -k_z z_r - \beta_z \dot{z}_r - m_2 \ddot{z}_1 \end{cases} \quad (10)$$

In this case, the column vector is \mathbf{Y} constructed to rewrite the above equation, and the construction \mathbf{Y} is as follows:

$$\mathbf{Y} = [y_1, y_2, y_3, y_4] = [z_1, z_r, \dot{z}_1, \dot{z}_r]^T \quad (11)$$

The above solution is still based on the assumption that the system will not be submerged ($\Delta V = \rho g \pi R^2 z_1$). The result of ode45, a function of MATLAB for solving ordinary differential equations, is shown in Figure 4 below:

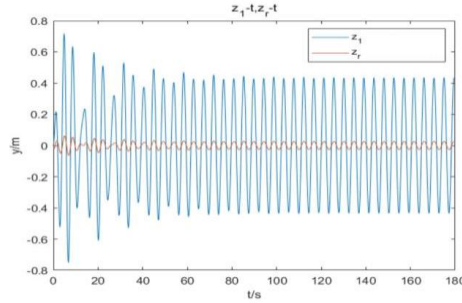


Figure 4: Diagrams of $z_1 - t$ and $z_r - t$ for linear dampers with constant damping coefficients

It can be seen that the float amplitude will not exceed 0.8m, so the system will not be submerged by water. It can also be observed that after about 100 seconds, the motion of the system becomes stable and enters into periodic motion. Using the relation $z_2 = z_1 + z_r$, the absolute velocity of the float and the oscillator can be calculated at the same time, and the chart is as shown in figure 5:

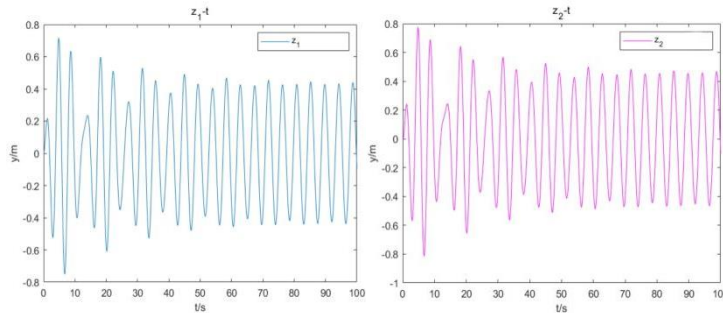


Figure 5: $z_1 - t$ and $z_r - t$ are shown in the first 100 seconds

Table 1 shows the heave displacement and velocity of the float and oscillator at 10s, 20s, 40s, 60s and 100s.

Table 1: Float and oscillator heave displacement and velocity at a specific time point

| t(s) | Float heave displacement (m) | Oscillator swing displacement (m) | Float speed (m/s) | Oscillator speed (m/s) |
|-------------|-------------------------------------|--|--------------------------|-------------------------------|
| 10 | -0.192 | -0.213 | -0.642 | -0.695 |
| 20 | -0.592 | -0.635 | -0.239 | -0.270 |
| 40 | 0.288 | 0.299 | 0.309 | 0.328 |
| 60 | -0.319 | -0.337 | -0.475 | -0.510 |
| 100 | -0.093 | -0.095 | -0.603 | -0.642 |

The damping coefficient of a linear damper is proportional to the power of the absolute value of the relative velocity, as shown in figure 6 below.

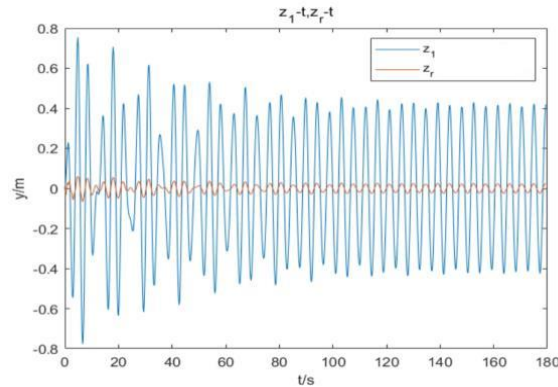


Figure 6: The damping coefficient of the linear damper and square root ($\sqrt{|Z_r|}$) is proportional to the $z_1 - t$ and $z_2 - t$ here

In this case, the float amplitude will also not exceed, so the system will not be flooded. In addition, after about 110 seconds, the motion of the system becomes stable and enters into periodic motion. The absolute velocity of the float and the oscillator can be calculated by using the equation $z_2 = z_1 + z_r$, as shown in Figure 7 and Table 2.

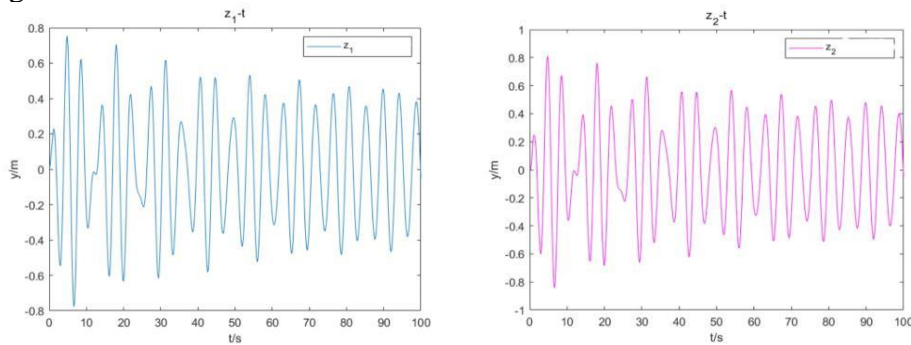


Figure 7: The first 100 seconds $z_1 - t$ and $z_2 - t$ the graph

Table 2 shows the heave displacement and velocity of the float and oscillator at 10s, 20s, 40s, 60s and 100s.

Table 2: Float and oscillator heave displacement and velocity at a specific time point

| $t(\text{s})$ | Float heave displacement (m) | Oscillator swing displacement (m) | Float speed (m/s) | Oscillator speed (m/s) |
|---------------|------------------------------|-----------------------------------|-------------------|------------------------|
| 10 | -0.192 | -0.213 | -0.642 | -0.695 |
| 20 | -0.592 | -0.635 | -0.239 | -0.270 |
| 40 | 0.288 | 0.299 | 0.309 | 0.328 |
| 60 | -0.319 | -0.337 | -0.475 | -0.510 |
| 100 | -0.093 | -0.095 | -0.603 | -0.642 |

2.6 Optimization of one-dimensional models

(1) The damping coefficient is constant

By constantly changing the initial value and the upper and lower bounds of the parameters, it is concluded that the power reaches the maximum value when the damping coefficient is $\beta_z = 3.7006 \times 10^4 \text{ N} \cdot \text{s}/\text{m}$, and its value is $P_{max} = 245.5386 \text{ W}$.

When β_z is within the range of $[3.7 \times 10^4, 5.0 \times 10^4]$, the change in P is not large, about $3W$. The above result is the optimal solution obtained by continuously enlarging and shrinking and changing the initial value.

(2) The damping coefficient is proportional to the power of the relative velocity

When the scaling coefficient and its power exponent are respectively $k = 100000.0000, c = 0.4026$, the power reaches the maximum value, which is $P_{max} = 278.100 \text{ W}$.

The above two cases will be completely submerged when the damping coefficient is small. But by the time it was submerged, the power had dropped to around 210W. Since the power function is continuously distributed, we believe that the highest power will not be taken at the edge of the parameter range.

3. The construction of two-dimensional model

In this paper, the two-dimensional case is considered, and a swing with a degree of freedom is added on the basis of the previous ones.

A central shaft hinged to the base by a rotating shaft is added, and the central shaft can be rotated around the rotating shaft. The PTO [6] system connects the oscillator to the rotating shaft and is located in the plane of the central shaft and the rotating shaft. A rotating damper and a torsional spring are symmetrically arranged on the rotating shaft to work together with the linear damper to output energy. So under the action of the wave the float in the cavity will rotate with the rotating shaft and it will vibrate with the rotating shaft and here the torque of the torsional spring is proportional to the relative angular displacement of the float and the oscillator; The torque of the rotary damper is proportional to the relative angular velocity of the float and the oscillator. The coordinate system in the two-dimensional case is shown in Figure 8.

In this paper, we only consider the motion of the hinged point on the Z-axis. Therefore, the motion of the float can be completely described by two parameters: the displacement $z_1(t)$ on the Z-axis and the angular displacement $\theta_1(t)$. Three parameters are needed to describe the motion of the oscillator: the Z-axis displacement $z_1(t)$, the Z-axis displacement $z_2(t)$ and the angular displacement $\theta_2(t)$.

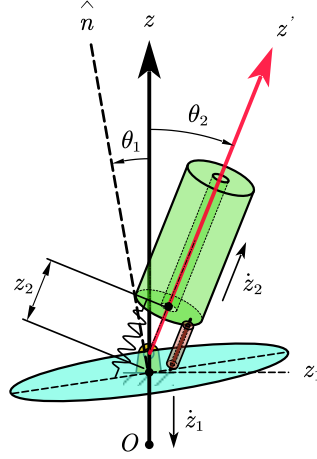


Figure 8: Coordinates in two dimensions

3.1 The establishment of a two-dimensional model

When considering the motion of the oscillator, its distance with respect to the rotating axis is always changing, so the moment of inertia of the oscillator with respect to the rotating axis cannot be defined directly. Here, the acceleration decomposition view in polar coordinates is used to investigate. The acceleration of a particle moving in the polar coordinate system is decomposed as follows:

$$\begin{cases} a_r = \ddot{r} - r\dot{\theta}^2 \\ a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} \end{cases} \quad (12)$$

Ignoring the mass of the central axis and the friction with the oscillator, the central axis is always in rotational equilibrium. Taking the hinged point as the rotating shaft, the torque on the central axis is investigated, and the following relationship can be found:

$$F_i d - \beta_\theta (\dot{\theta}_2 - \dot{\theta}_1) - k_\theta (\theta_2 - \theta_1) = 0 \quad (13)$$

Where, d in the above formula represents the moment arm of the supporting force, and its magnitude is expressed as follows:

$$d = l_0 - \frac{m_2 g}{k_z} + \frac{h}{2} + z_2 \quad (14)$$

Then considering the stress of the oscillator. Oscillators are subjected to inertial forces, gravity, supportive forces, and PTO. It should be noted that after the spring is tilted, the gravity that was originally completely along the rod is no longer along the rod, resulting in an extra $mg(1 - \cos(\theta_2))$ force acting along the rod. Along the rod direction, with outward as positive, there are:

$$-m_2 \ddot{z}_1 \cos \theta_2 + m_2 g (1 - \cos \theta_2) - (k_z z_2 + \beta_z \dot{z}_2) = m_2 \ddot{z}_2 - m_2 d \cdot \dot{\theta}_2^2 \quad (15)$$

Then list the tangential equation:

$$-F_i + m_2 g \sin \theta_2 + m_2 \ddot{z}_1 \sin \theta_2 = m_2 (2\dot{z}_2 \dot{\theta}_2 + d \cdot \ddot{\theta}_2) \quad (16)$$

3.2 The establishment of a two-dimensional model

The two ends of the damper are respectively followed by the base and the oscillator, so the relative velocity along the Z-axis should be considered when calculating the work [7] done by the damper. Therefore, the work done on the linear damper in time t is:

$$W_z(t) = \int_0^{t_s} \beta_z \dot{z}_r dz_r \quad (17)$$

Similarly, a similar formula can be written for a rotating damper:

$$W_\theta(t) = \int_0^t \beta_\theta (\dot{\theta}_2 - \dot{\theta}_1)^2 dt \quad (18)$$

From this, the average power in time t can be written. For the first two questions, just take $W_\theta(t) = 0$

$$P = \frac{W_z(t) + W_\theta(t)}{t} \quad (19)$$

3.3 Optimization problem in two dimensions

Solving the two-dimensional case has the following five constraint equations [8]:

$$\left\{ \begin{array}{l} F_i d - \beta_\theta (\dot{\theta}_2 - \dot{\theta}_1) - k_\theta (\theta_2 - \theta_1) = 0 \\ -m_2 \ddot{z}_1 \cos \theta_2 + m_2 g (1 - \cos \theta_2) - (k_z z_2 + \beta_z \dot{z}_2) = m_2 \ddot{z}_2 - m_2 d \cdot \dot{\theta}_2^2 \\ -F_i + m_2 g \sin \theta_2 + m_2 \ddot{z}_1 \sin \theta_2 = m_2 (2\dot{z}_2 \dot{\theta}_2 + d \cdot \ddot{\theta}_2) \\ (m_1 + m_a) \ddot{z}_1 = f \cos \omega t - \rho g \pi R^2 z_1 - \beta_{ez} \dot{z}_1 + (k_z z_2 + \beta_z \dot{z}_2) \cos \theta_2 + m_2 g (1 - \cos \theta_2) \\ L \cos \omega t - \beta_{e\theta} \dot{\theta}_1 - \beta_b \theta_1 + m_1 \ddot{z}_1 L_c \sin \theta_1 = (J_a + J_1) \ddot{\theta}_1 \end{array} \right. \quad (20)$$

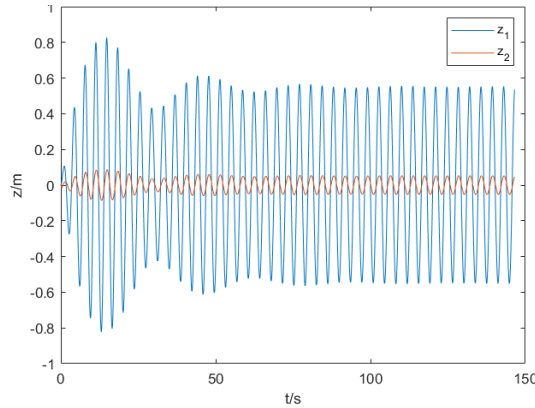


Figure 9: Float and oscillator displacement - time diagram

It can be seen from figure 9 above that the amplitude of the float still does not exceed 1m, so there

will be no submersion. It can also be seen that the spring deformation is not very large, and the system tends to be stable after about 60s. Figure 10 shows the relationship between the angular displacement of float and oscillator with time:

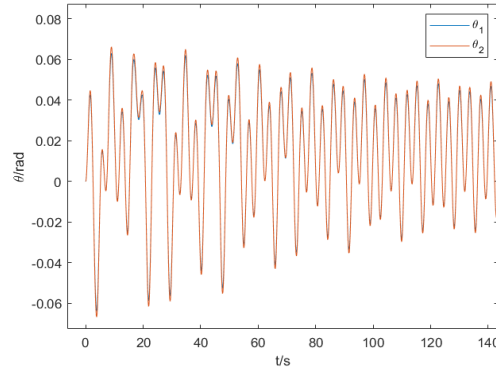


Figure 10: Float and oscillator angular displacements - time diagram

The angular displacement is shown in Figure 11, and the Angle difference between the float and the oscillator is quite obvious:

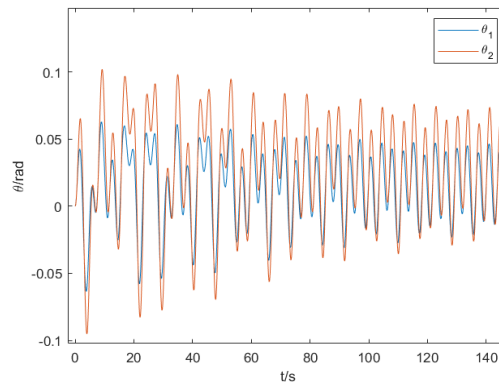


Figure 11: Float and oscillator displacements - time diagram (stiffness coefficient of torsional spring is 0.1 times of original)

4. Conclusion

In this paper, the model is simplified appropriately on the basis of practice, and the calculation amount is effectively reduced by using appropriate assumptions, and the physical derivation is rigorous. The motion condition obtained has reference significance for practice. In the aspect of optimization, sqp algorithm is used to efficiently solve the extreme values of univariate and multivariable, and the traversal search algorithm is introduced. First, it uses the rough analysis of big step length, and then the exact analysis of small step length, which reduces the calculation amount, avoids the program ending in the local optimal solution, and makes the results more accurate. The reality is far more complex than the model. There are actually degrees of freedom in other directions, and the abscissa of the hinge point is going to change over time.

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