# Study on Excavation Period Estimation of Deep Hole Presplitting Blasting Based on Monte Carlo

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*Abstract:* In order to improve the tunneling efficiency of hard rock roadway, its construction process is better controlled. The deep hole presplitting blasting excavation process of 1605 roadway in Guhanshan Mine is taken as the object. The Oracle Crystal Ball software is used to simulate the process of roadway excavation by Monte Carlo simulation. The possible construction period time and distribution law of single cycle of roadway excavation are predicted. The sensitivity of each process is analyzed, and the bottleneck process that has a great influence on the process is found. Then, the process that needs to be controlled is further analyzed and studied. The drilling depth of the drilling process is changed, and the tunneling speed and the change of sensitivity of each process under different drilling depths are analyzed. The results show that the Monte Carlo simulation method can calculate the duration of roadway excavation, find the bottleneck process of roadway excavation process, and find the optimal parameters of its process.

# 1. Foreword

Floor gas extraction is one of the main technical means of gas control in outburst coal seam. The tunneling efficiency and surrounding rock control of bottom drainage roadway restrict the efficiency of gas extraction, which will have a great impact on mining replacement. In order to facilitate the maintenance and installation of the extraction system, the bottom extraction roadway is mostly selected in relatively stable rock strata, such as sandstone with high strength. In hard rock excavation, prefabricated fracture methods such as shallow hole blasting, ultra-deep hole blasting, static fracturing and hydraulic fracturing are often used to form artificial fracture loose surrounding rock in rock mass, and then tunneling is carried out by roadheader. This method has the problems of long operation time and low efficiency, which can not meet the requirements of roadway excavation<sup>[1-3]</sup>. Therefore, before construction, a scientific and reasonable construction plan should be formulated to find the key processes that affect the overall situation, determine the better construction parameters, reduce costs and improve efficiency.

Due to the influence of human resources, machinery and equipment and other factors, the time of

each process is uncertain in the process of roadway construction and excavation. Monte Carlo simulation is a random simulation technology widely used in various fields<sup>[4]</sup>. The simulation technology takes probability theory and mathematical statistics as the core, and randomly calculates the established model, which has a large number of applications in project schedule management. Wang Weidong et al.<sup>[5]</sup> simulated the process and total construction period of high-speed railway track slab laying by Monte Carlo method, and determined the probability distribution of construction period and the interaction between processes. Liu Bing et al. <sup>[6]</sup>carried out Monte Carlo simulation on the progress of space science engineering projects, speculated the completion time of engineering projects, and conducted sensitivity analysis. Li Tao et al.<sup>[7]</sup> combined Monte Carlo simulation and network disjoint method to calculate the reliability of construction period, cost, quality and project construction. Taking a nuclear power project as the object, Li Guoying<sup>[8]</sup> realized the risk assessment of the total construction period of the project through Monte Carlo simulation. Based on the traditional critical path method, Huang Yu<sup>[9]</sup> estimated the installation period of offshore platform by using Monte Carlo method. Taking tram pavement as the object, Liu Jun et al. <sup>[10]</sup> used Monte Carlo method to simulate the total construction period of tram track structure laying, and used building information model technology to achieve construction management informatization. Jia Guangshe et al.<sup>[11]</sup> analyzed the construction progress data of many large airports in China, determined the process sequence of typical critical routes of the terminal, fitted the time distribution of each process, and established the estimated construction period model of the terminal through Monte Carlo method.

In summary, a large number of researchers have used the Monte Carlo method to explore different engineering fields, and have made some progress. It provides a certain reference for engineering practice in terms of duration estimation, engineering risk assessment, and engineering reliability assessment. However, it is less applied in the field of mining project duration management and optimization, and when a certain parameter changes, there is less research on its impact on the duration. In view of this, based on the example of deep hole pre-splitting blasting process in 1605 roadway of Guhanshan Mine, this paper uses Monte Carlo simulation to analyze and study it. After using Monte Carlo method to simulate the construction period, this paper discusses the influence of parameter changes of drilling process on the construction period, in order to provide basis for its engineering project management from the perspective of management, and provide reference for the management and control optimization of similar projects in mining industry.

# 2. Tunneling technology

#### 2.1 Engineering background

The 1605 bottom drainage roadway of Guhanshan Mine is located in the sandstone layer of the floor. The hard rock of this layer is siltstone, gray, powdery, mainly quartz, followed by feldspar, cementation, and the Platts coefficient reaches 9-11. In the process of roadway excavation, there are some problems such as low efficiency of comprehensive excavation, poor control of construction process, and difficulty in determining some construction parameters. It is necessary to further optimize and control the comprehensive excavation process and parameters to alleviate the contradiction between mining and excavation.

#### 2.2 WBS decomposition

There are many steps in the construction scheme of deep hole pre-splitting blasting in roadway. It is necessary to decompose the construction process layer by layer according to reasonable basis, and make detailed construction plan to ensure that the roadway excavation meets the requirements of the specification. The Work Breakdown Structure method is based on scientific, reasonable and logical

requirements, and groups the project plan. The process after grouping can judge the project completion faster and clearer, making project management more convenient. After investigating the excavation site of 1605 working face in Guhanshan Mine in detail, this paper decomposes a cycle of roadway deep hole pre-splitting blasting excavation technology into 12 procedures based on WBS method, as shown in Tab 1.

Task Number	Job Title	Predecessor Lag I		Duration/min	
1	Handover,Safety confirmation		2	10	
2	Lead, Draw contour line	1	3	10	
3	Cleaning working face, Calibration hole position	2	4	10	
4	Bore hole	3	5	90	
5	Charge, Connection, sealing	4	6	35	
6	Detonation	5	7	15	
7	Site cleaning, Safety confirmation	6	9	30	
8	Maintenance boring machine, Gangue preparation		9	20	
9	Roadheader cutting, Discharging gangue	7, 8	10	1500	
10	Interrogation, Safety confirmation	9	11	10	
11	Temporary support	10	12	30	
12	Field cleaning	11	13	10	

# 3. Network graph

According to the construction process of roadway deep hole pre-splitting blasting in Figure 1, the conditions of project construction are determined on the spot, 30 groups of duration sample values are collected for each process, the duration range is calculated, and the project plan table of single cycle of roadway excavation is obtained. The plan table points out the tight front and tight back processes of each process according to the relationship between the front and back of Fig 1, and selects the ideal estimation of the duration of each process according to the duration range. According to Tab 1, a double-code time-scale network plan diagram is made<sup>[12-16]</sup>, which can provide a model basis for subsequent simulation calculations. From Fig 1, we can clearly see the logical method of calculating the construction period of a single cycle. It can be seen that the tunneling machine cutting, gangue and drilling process last longer than other processes, which should be paid more attention to.



Figure 1: Network plan of double code time of roadway deep-hole pre-splitting blasting excavation

# 4. Monte Carlo simulation

In this paper, based on the data collected in the roadway excavation site, the probability model and related parameters of the duration of each process are fitted and determined<sup>[17]</sup>. According to the roadway excavation model established by the previous plan review technology, the Monte Carlo simulation test is used to obtain the probability distribution of the construction period of a cycle of roadway excavation when the borehole is 25 m.

# 4.1 Mathematical distribution of process duration

In the process of roadway excavation construction, the duration of each procedure is often related to the on-site engineering resources and possible risk factors, which will change. Through on-site investigation, 30 sample values of the duration of each process were obtained when the 1605 roadway of Guhanshan Mine was drilled by deep hole pre-splitting blasting for 25 m.

Using Oracle Crystal Ball software to distribute and fit the collected 30 sets of data of the duration of each process, the fitting of the duration of 12 processes of deep hole pre-splitting blasting excavation at 25 m is obtained. It can be seen from Tab 2 that the probability distribution of each process after fitting is mainly normal distribution and triangular distribution.

Task	Job Title	Number of	Probability distribution types
Number	Job The	Samples	and parameters /min
1	Handover,Safety confirmation	30	N(10.1,2.2 <sup>2</sup> )
2	Lead, Draw contour line	30	N(9.8,3.6 <sup>2</sup> )
3	Cleaning working face, Calibration hole position	30	N(10.2,6.3 <sup>2</sup> )
4	Bore hole	30	T(62.1,90.7,108.2)
5	Charge, Connection, sealing	30	N(33.1,5.6 <sup>2</sup> )
6	Detonation	30	N(15.2,6.6 <sup>2</sup> )
7	Site cleaning, Safety confirmation	30	N(29.5,3.7 <sup>2</sup> )
8	Maintenance boring machine, Gangue preparation	30	N(19.7,2.1 <sup>2</sup> )
9	Roadheader cutting, Discharging gangue	30	T(1 349.2,1 502.2,1 647.9)
10	Interrogation, Safety confirmation	30	N(10.7,2.5 <sup>2</sup> )
11	Temporary support	30	N(31.5,6.2 <sup>2</sup> )
12	Field cleaning	30	N(10.3,2.5 <sup>2</sup> )

Table 2: The duration fitting of each process

(N is normal distribution, T is triangular distribution)

#### **4.2 Simulation process**

Oracle Crystal Ball software is an extension tool of excel vba. The model is established according to the logical relationship of Fig.2, and the process distribution generated by fitting is imported into the hypothesis unit. During the simulation calculation, the data is randomly generated in the hypothesis unit according to the fitting situation. The end time of a single cycle is imported into the prediction unit.

Set the hypothesis unit and the prediction unit, set the number of simulations, and start the simulation operation. In each simulation, a set of data is formed according to the probability of fitting. According to the formula, the simulation system will collect the data of each hypothesis unit and prediction unit for subsequent analysis.

1	A	В	С	D	E	F	G	н	1	J	К	L
2	Job Title	Task number	Predecessor	Duration Pessimisticu	Most likely	Most optimistic	Sim Value	EST	EFT	LST	LFT	Slack
3	Handover, safety confirmation	Α	Si	=H20+I20	=H20	=H20-I20	10	0	=H3+G3	=J4-G3	=J3+G3	=J3-H3
4	Lead, draw outlines	В	Α	=H21+I21	=H21	=H21-I21	10	=I3	=H4+G4	=J5-G4	=J4+G4	=J4-H4
5	Clean working face, calibrate the hole position	C	В	=H22+I22	=H22	=H22-I22	10	=I4	=H5+G5	=J6-G5	=J5+G5	=J5-H5
6	Bore hole	D	С	=E6*'parameter setting	=H23*'para	=E6*'parameter	90	=15	=H6+G6	=J7-G6	=J6+G6	=J6-H6
7	Charge, wire, seal hole	E	D	=H24+I24	=H24	=H24-I24	35	=16	=H7+G7	=J8-G7	=J7+G7	=J7-H7
8	Detonation	F	E	=H25+I25	=H25	=H25-I25	15	=I7	=H8+G8	=J9-G8	=J8+G8	=J8-H8
9	Site cleaning, Safety confirmation	G	F	=H26+I26	=H26	=H26-I26	30	=I8	=H9+G9	=J10-G9	=J9+G9	=J9-H9
10	Maintenance boring machine, Gangue preparation	Н	G	=H27+I27	=H27	=H27-I27	20	=I9-10	=H10+G10	=J11-G10	=J10+G10	=J10-H10
11	Roadheader cutting, Discharging gangue	I	GH	=E11*'parameter settir	=H28*'para	=E11*'paramete	1500	=MAX(I9:I10)	=H11+G11	=J12-G11	=J11+G11	=J11-H11
12	Interrogation, Safety confirmation	J	I	=H29+I29	=H29	=H29-I29	10	=I11	=H12+G12	=J13-G12	=J12+G12	=J12-H12
13	Temporary support	K	J	=H30+I30	=H30	=H30-I30	30	=I12	=H13+G13	=J14-G13	=J13+G13	=J13-H13
14	Field cleaning	L	K	=H31+I31	=H31	=H31-I31	10	=I13	=H14+G14	=I14	=J14+G14	=J14-H14
15									=114			

Figure 2: Oracle Crystal Ball model diagram

# 4.3 Simulation analysis

According to the fitting situation of tab 2, using Oracle Crystal Ball software, the model is established by the logical relationship and time parameters between the work shown in tab 1, and the hypothesis unit and prediction unit of the model are defined by the time probability distribution of each process shown in tab 2. The Monte Carlo simulation of deep hole pre-splitting blasting excavation at 25 m of 1 million boreholes is carried out, and 1 million results of 25 m deep hole pre-splitting blasting excavation are determined. The analysis of the results is as follows :

(1) The duration frequency distribution of a single cycle and the fitted probability distribution are shown in Fig.3 (a). It can be seen from Fig.3 (a) that the probability distribution of single cycle duration of 25 m deep hole pre-splitting blasting excavation is more in line with Beta distribution. The shortest duration is 1 556.81 min, and the longest duration is 1 949.17 min. The probability peak corresponds to 1 757.10 min, and the average value is 1 756.79 min.

(2) The simulation results of the cumulative probability distribution of the duration of a single cycle are shown in Fig.3 (b). It can be seen from Fig.3 (b) that the probability of completion within 1 930 min is 99.98 %, the probability of completion within 1 850 min is 92.40 %, and the probability of completion within 1 800 min is 74.04 %.







(b) Duration cumulative probability distribution

Figure 3: Simulation duration probability and cumulative probability

(3) The sensitivity of each process is shown in Fig.4.Sensitivity is the degree of influence on the overall project when the process changes. It can be seen from Fig.4 (a) that the cutting and gangue discharge process of roadheader has the greatest influence, up to 93.65 %, so it is necessary to focus on controlling the tunneling process. The tunneling process is the most important process in the whole construction process, and its sensitivity is too high. Fig.4 (a) does not see the degree of mutual influence of other processes. Without considering the tunneling process, the sensitivity is recalculated, as shown in Fig.4 (b). The sensitivity of the drilling process is 31.54 %, followed by the four processes of temporary support, examination of the roof, blasting preparation and detonation, all of which are above 10 %. In the construction process, in addition to paying attention to the tunneling process, we should also focus on the drilling process to reduce its impact on the construction period.



Figure 4: Sensitivity analysis

#### 5. Simulation under different drilling depths

#### **5.1 Drilling process control**

From Figure 3, it can be seen that the sensitivity of the two processes of cutting, gangue discharge and drilling of the roadheader is high, but the cutting and gangue discharge of the roadheader are limited by factors such as mechanical equipment and on-site environment. In the simulation process, the duration of the single process has been converted into a probability function. Taking some factors into account, it is impossible to further quantify the control. The drilling process also takes some factors into account, but the depth of the drilling determines the total excavation period of a cycle. When the depth of the drilling blasting is changed, the excavation time per meter of the construction period will change, and the change rule is found out, which can provide further reference for the construction schedule control.

# 5.2 Drilling depth range

Deep hole pre-splitting blasting excavation is to form artificial fracture loose surrounding rock in rock mass, and then use roadheader to excavate. When the explosive explodes, the rock mass is broken loose but not thrown. The internal action principle of pre-splitting blasting is shown in Fig.5 Fig.5 (a) is a schematic diagram of an explosive hole. The deep hole explosive makes it form a crushing ring, a vibrating ring and a fissure ring. Fig.5 (b) is a reasonable design of the hole distribution of the deep hole blasting in the rear roadway. After the three blast holes burst loose rock, the use of roadheader tunneling can achieve the purpose of improving the tunneling speed.



Figure 5: Internal action principle of pre-splitting blasting

In the figure: R<sub>k</sub>-Cavity radius; R<sub>c</sub>-Crushing ring radius; R<sub>p</sub>-Fracture circle radius.

Taking Guhanshan Mine as an example, the hole depth L is determined according to the nature of surrounding rock and the blasting effect of explosives, and is calculated by formula (1).

$$L = \frac{W+h}{\sin\theta} \tag{1}$$

In the formula W——Critical depth of charge,m;

*h*——Reserved complete rock mass thickness,m;

 $\theta$ ——Sharp angle between blast hole and roadway floor,(°).

According to the theory of explosion stress wave, *W* is calculated to be 7.54 m, and the thickness of complete rock mass *h* is 1.2 m. Then  $W + h \approx 10$ , and the range of borehole depth is as follows : (2).

$$L = \frac{10}{\sin \theta}$$
(2)

Therefore, the minimum depth is 10 m. Considering the mechanical equipment, blasting effect and safety, the longest hole depth is 45 m.

#### **5.3 Probability distribution of different drilling time**

The working method of drilling deep holes is mainly to increase the drilling depth by continuously increasing the drill pipe from the rear of the drilling rig. After field investigation, when the hole depth is different, the probability distribution of the drilling process is still triangular, the parameters of the triangular distribution change linearly, the tunneling process is triangular, and the parameters also change linearly. The probability distribution and parameters of the remaining processes will not change significantly. Among them, the workload of temporary support will change with the change of drilling depth, but it will work in parallel with the tunneling and gangue discharge process of roadheader. Therefore, its probability distribution and parameters are considered as unchanged. The probability distribution type of each process duration and the change of parameters are shown in tab 3. In order to facilitate the calculation, the parameters are rounded by approximate method.

Task Number	Job Title	Probability distribution types and parameters/min	variation
1	Handover,Safety confirmation	N(10,2 <sup>2</sup> )	No
2	Lead, Draw contour line	N(10,4 <sup>2</sup> )	No
3	Cleaning working face, Calibration hole position	N(10,6 <sup>2</sup> )	No
4	Bore hole	T(2.52 <i>l</i> ,3.6 <i>l</i> ,4.32 <i>l</i> )	Linear Change
5	Charge, Connection, sealing	N(35,5 <sup>2</sup> )	No
6	Detonation	$N(15,6^2)$	No
7	Site cleaning, Safety confirmation	$N(30,4^2)$	No
8	Maintenance boring machine, Gangue preparation	N(20,2 <sup>2</sup> )	No
9	Roadheader cutting, Discharging gangue	T(54 <i>l</i> ,60 <i>l</i> ,66 <i>l</i> )	Linear Change
10	Interrogation, Safety confirmation	N(10,2 <sup>2</sup> )	No
11	Temporary support	N(30,6 <sup>2</sup> )	No
12	Field cleaning	N(10,2 <sup>2</sup> )	No

Table 3: Parameter change	ze	2
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In the table, *l* is the selected drilling depth during tunneling.

#### 5.4 Different drilling depth simulation and analysis

Based on the Monte Carlo simulation scheme for deep-hole pre-splitting blasting excavation when the borehole is 25 m, a million Monte Carlo simulations are performed using the assumed and predicted units of the probability distribution and parameter definition model of each process that are consistent with expectations from 10 m to 45 m with an interval of 1 m. The results are analyzed as follows:

(1) When the drilling depth (l) is changed, the probability distribution of fitting time is in accordance with Beta distribution. The tunneling speed (v) of different drilling depths can be obtained by selecting the mathematical expectation of Beta distribution simulated by different drilling depths as a reference. The change is shown in Fig. 5. It can be seen from Fig.5 that as l increases, v gradually decreases. When l = 10 m, v = 80.58 min/m. When l = 45 m, v = 67.32 min/m. From 10 m to 45 m, the tunneling speed is increased by about 13 min/m, and the efficiency is improved. The reason is that as l increases, the time proportion of the two key processes of drilling and tunneling in one cycle is getting higher and higher, while the remaining processes are affected by the nature of work, and the duration will not change, and the proportion is getting smaller and smaller. The curve is concave as a whole. As 1 increases, the slope becomes smaller and smaller, and the effect of increasing the tunneling speed becomes smaller and smaller. The reason is that in addition to drilling and tunneling processes, the influence of other processes is getting smaller and smaller. Decision makers can select a longer drilling depth according to the actual situation to obtain a faster tunneling rate. The Monte Carlo simulation results in this paper can provide decision-making reference.



Figure 6: Driving speed of different drilling depth

(2) When changing 1, the sensitivity of each process corresponding to different drilling depths changes as shown in fig.7. The sensitivity of the tunneling process increases rapidly from 79.17 % to 95.2 %, and then increases slowly. The sensitivity of other processes except tunneling and drilling processes decreased rapidly from 19.55 % to 2.7 %, and then decreased slowly to 2.7 %. The drilling process sensitivity changes little, but also slowly rising from 1.28 % to 2.2 %. This corresponds to the change in Fig.6, which is why the slope in Figure 5 is getting smaller. With the increase of 1, the degree of dispersion of the remaining process, and the sensitivity of the tunneling process increases. More time occupation is inclined to the tunneling process, and the sensitivity of the tunneling process increases. With the increase of 1, the drilling time will increase but the proportion is relatively low, so the sensitivity of the drilling process will rise slowly.

When increasing the drilling depth, the focus of the work of the project tend to tunneling process, but with the increase of l, the possibility of blasting<sup>[18]</sup>, mechanical tunneling process risk will increase, and too much increase the drilling depth, the effect of improving efficiency is low. Therefore, in the actual application process, it is not necessary to select a deeper drilling depth to achieve the purpose of improving efficiency. According to the actual situation, a more efficient drilling depth should be selected in the case of lower risk.



Figure 7: Variation of process sensitivity

# 6. Conclusion

Taking the excavation process of 1605 bottom drainage roadway in Guhanshan Mine as the research object, the Monte Carlo simulation was used to calculate the excavation period of deep hole pre-splitting blasting and various conditions under different drilling depths. The conclusions are as follows:

(1) Through the analysis of 30 sets of data from the field investigation, the fitting distribution of the duration of each excavation process was determined. Through Monte Carlo simulation, it is determined that the duration of a single cycle of 25 m deep hole pre-splitting blasting excavation obeys Beta distribution, and the duration is about  $1750 \sim 1950$  min. The change of each process has different influence on the construction period, and the two processes of tunneling and drilling should be controlled.

(2) The average construction period of different drilling depths is simulated by Monte Carlo simulation method. It is determined that the higher the drilling depth is, the higher the tunneling speed is. According to the sensitivity analysis and multi-factor consideration, the selection principle of the best drilling depth is determined.

(3) Monte Carlo simulation method is a convenient method suitable for roadway deep hole presplitting blasting excavation construction schedule management and parameter optimization, which can provide basis for controlling construction period and compiling construction plan.

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