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Research of Option Pricing Application Based on Black-Scholes and Binary Tree Model

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Abstract: Option pricing is very important for investors' option transactions in the financial market. This article introduces the Black-Scholes model and the binary tree model, and the theoretical pricing of a European call option under risk-neutral conditions is solved by the two models. The theoretical pricing is compared with the actual option closing price. The results show that these two models have higher precision and better effect on the theoretical pricing of options.

1. Introduction

Options are the most frequently traded financial derivatives in the financial market. Pricing theory has a great influence on the financial market. Therefore, many scholars are keen to study option pricing methods. Since the birth of options, people have discovered that during the trading process, the price of an option is partly affected by the price of the underlying asset, as well as some other factors [1]. Under normal circumstances, traders and investors generally use different mathematical models to calculate the price of options, and the use of option pricing models can enable option investors to trade rationally and avoid blind speculation. For ordinary investors, the Black-Scholes (B-S) option model is simple and easy to understand. Li Yiwei used the B-S model and the binary tree model to price CIS 300 stock index option. Wu Anlin and Li Xing used the B-S model and binary tree model for comparative analysis. Some scholars used the binary tree method to study option pricing. This article takes CSI 300ETF options of Shanghai Stock Exchange as the research object, and corresponding underlying asset of this option is the stock 510300.The annual volatility is obtained by counting historical data of the stock. The binary tree and B-S model are used to analyze this stock option. The theoretical option pricing is obtained by Matlab and is analyzed.

2. Black-Scholes Model

The Black-Scholes model is an accurate model for option pricing proposed by two economists. The model has the following assumptions: (1) The price of stocks obeys a lognormal distribution (2) The risk-free interest rate is constant (3) There is no arbitrage opportunity in the financial market (4) The option is a European option and will not be executed before expiration (5) The stock does not pay dividends (6) There are financial assets that can be traded continuously.

Based on the above assumptions we can derive the pricing formula of European call options, the underlying assets refer to stock prices. The random process of the underlying asset is:

$$dS = \mu S dt + \sigma S dW \qquad (1)$$

Where S is the stock price, μ is the average value of the stock price, σ is the stock price volatility (standard deviation) ,dW is a Wiener process.

Suppose f is the derivatives price based on S.According to the theorem of Ito, the differential equation is obtained:

$$\frac{\sigma f}{\sigma t} + rS\frac{\sigma f}{\sigma S} + \frac{1}{2}\sigma^2 S^2 \frac{\sigma^2 f}{\sigma S^2} = rf \qquad (2)$$

The boundary condition of European call options is $f = \max(S - X, 0)$. When $t = T$, using partial differential equations to solve the above formula, the European call option pricing formula is obtained as:

$$c = SN(d_1) - Xe^{-rt}N(d_2) \qquad (3)$$

Where: $d_1 = \frac{\ln(\frac{S}{X}) + (r + \frac{\sigma^2}{2})t}{\sigma t}$ □ , $d_2 = d_1 - \sigma\sqrt{t}$, X is strike price, t is the remaining year of the stock option, c is the value of European call options(yuan), $N(\ )$ is the standard normal distribution function, σ represents the annual volatility of the stock, r is risk-free rate.

The calculation of σ is as follows:

$$\sigma = \sigma_1\sqrt{annual\ trading\ days} \qquad (4)$$

Where: $\sigma_1 = \sqrt{\frac{1}{T\text{-}1}\sum_{i=1}^{T}(R_i - \mu)^2}$, $R_i = \ln(\frac{P_i}{P_{i-1}})$, P_i is the stock closing price on the i day(yuan), R_i is the logarithmic return rate of P_i, μ is the average of R_i, T indicates the trading days of the stock, σ_1 is the variance of R_i.

3. Binomial Tree Model

The binary tree model was first proposed by Cox, Ross, and Rubinstein in 1979, which used bifurcated branches to describe the change process of stock and option prices. That is to say, there are only two possibilities for stock prices to rise and fall at the next point in time. Figure 1 shows the multi-step binary tree of stock prices.

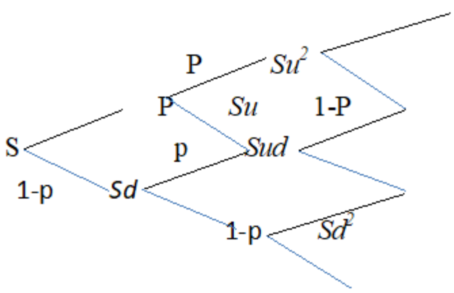

Fig.1 Binary Tree Model Diagram

Assume that the duration of the option is t. Divide the time interval [0, t] into N equal intervals, length is $\Delta t = \frac{t}{N}$. Assume that the initial price of the stock price is S, at time Δt the price rises to Su with probability p or falls to Sd with probability $1\text{-}p$. According to literature, $u = \frac{1}{d}$. At time $2\Delta t$, the stock price may have three situations: Su^2, Sud and Sd^2. At time $3\Delta t$, the stock price may have four situations: Su^3, Su^2d, Sud^2, Sd^3.By analogy, at time $i\Delta t$, the stock price corresponding to the node (i, j) is Su^jd^{i-j}, and the corresponding option price is fi, j. Consider a European call option with no dividend payment, the stock price at expiration t is Su^jd^{N-j}, and the corresponding option price is $f_{N,j}$:

$$f_{N,j} = \max(Su^jd^{N-j} - K,0) \quad j = 0,1,2,\cdots N \tag{5}$$

Where K represents the strike price of the option

From the risk-neutral valuation:

$$f_{i,j} = e^{-r\Delta T}[pf_{i+1,j+1} + (1-p)f_{i+1,j}],0 \le i \le N-1,0 \le j \le i \tag{6}$$

Where $u = e^{\sigma\sqrt{\Delta t}}$, $d = \frac{1}{u}$, $p = \frac{e^{r\Delta t} - d}{u - d}$

Use this recurrence formula(6)to calculate the option price f0,0 at the initial moment.

4. Analysis of Option Pricing Examples

The CSI 300ETF option is a European option, which has recently listed in China. This article takes 300ETF's call option of March as an example. The expiration date is March 24, 2021 and the strike price is 4.836 yuan. Daily closing price data of the 510,300 stock are selected from September 17, 2020 to February 22, 2021 in the Big Wisdom Data. Part of the daily closing price data and the calculated logarithmic return rates are shown in Table 1. Assume that the number of trading days in China's stock market is 246 days per year and the annual stock volatility is 0.194447 according to formula (4).

Table 1 510300 Stock Sample Data

Time	P_i (yuan)	R_i
2020/9/17	4.618	
2020/9/18	4.739	0.025864
2020/9/21	4.684	--0.01167
2020/9/22	4.627	--0.01224
2020/9/23	4.641	0.003021
2020/9/24	4.564	-0.01673
…		
2021/2/19	5.796	0.005536
2021/2/22	5.589	-0.03637

The annualized fixed deposit interest rate stipulated by the Central Bank is the risk-free rate, so the risk-free rate r selects the one-year fixed deposit rate of 1.5%. According to different time to the expiration date of the option, September 24, 2020, October 23, 2020, November 24, 2020, December 24, 2020, January 25, 2021and 2021 February 25th is selected for the option trading day. The B-S model and the binary tree model are used to calculate the options at these times by

Matlab. The pricing of the options and the relative error of options are shown in Table 2. The option daily closing prices are obtained from the big wisdom software. The relative error tween B-S theoretical pricing c and option daily closing price is calculated as follows:

$$\delta_1 = (c - c_2) / c_2 \tag{7}$$

Where c_2 is option daily closing price (yuan)

The relative error δ_2 of options between binary tree and option daily closing price is calculated as follows:

$$\delta_2 = (f_{0,0} - c_2) / c_2 \tag{8}$$

Table 2 Pricing of The Options and the Relative Error of Options

Time	B-S	δ_1	N	Binary tree	δ_2	c_2
2020/9/24	0.1813	-2.74%	10	0.1872	0.43%	0.1864
			50	1807	-3.06%	
			100	0.1819	-2.41%	
2020/10/23	0.2227	6.91%	10	0.2235	7.3%	0.2083
			50	0.2238	7.44%	
			100	0.2233	7.20%	
2020/11/24	0.3249	6.14%	10	0.3299	7.78%	0.3061
			50	0.3243	5.95%	
			100	0.3254	6.31%	
2020/12/24	0.2923	5.87%	10	0.2965	7.39%	0.2761
			50	0.2927	6.01%	
			100	0.2925	5.94%	
2021/ 1/25	0.745	-1.02%	10	0.7441	-1.14%	0.7527
			50	0.7449	-1.04%	
			100	0.7449	-1.04%	
2021/2/25	0.5987	-3.82%	10	0.5984	-3.87%	0.6225
			50	0.5987	-3.82%	
			100	0.5987	-3.82%	

The binary tree model is an approximation of the Black-Scholes model in discrete time. When the number of steps N is larger, the calculated option pricing is closer to the value calculated by the B-S model. There is a certain error between option pricing and option closing price, which may be due to the assumptions made by the model. The actual option market is more complicated.

5. Conclusion

The theoretical pricing of CSI 300ETF call options is carried out by B-S and binary tree model, and the calculation is carried out by Matlab. The theoretical pricing of options is relatively close to the closing price of options, and more reasonable theoretical values are obtained. The B-S model is only suitable for European options, and the binary tree model can also be used for American options. Both the B-S model and the binary tree model can be used for the prediction of CSI 300ETF call options and be suitable for the prediction of put options, which has certain guiding significance for the pricing of 300ETF options.

References

[1] Lucy F Ackert. Efficiency in index options markets and trading in stock baskets. Journal of Banking and Finance, no.09, pp.1-4, 2001.