Evaluation model of higher education based on entropy weight method

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Abstract: With the gradual expansion of higher education in recent decades and the response to emergencies such as epidemics, all countries need to think about which of their higher education systems is effective and which can be better. How to measure and assess the health of higher education systems at the national level is among the most far-reaching research topics in the world today. This paper makes an in-depth analysis of the above problems and draws some valuable conclusions. First of all, we build a model: entropy weight TOPSIS method to evaluate the higher education system of each country, and establish a comprehensive evaluation index system. Select part of the higher education development level index for data collection, and then select two typical developed countries and developing countries to evaluate, using entropy method to calculate the weight of each index. Finally the TOPSIS method is adopted to evaluate the higher education system in these countries, and rank each country according to its evaluation scores.

1. Introduction

The higher education system is an important part of a country's efforts to further educate its citizens beyond primary and secondary education. Therefore, as a source of trained and educated citizens for the country's economic development, the higher education system has value.

In order to measure and evaluate the health of the higher education system at the national level and determine a healthy and sustainable state for the higher education system of a given country, we will use entropy weight TOPSIS method to develop and validate a model or a set of models to evaluate the higher education system of each country.

2. Model establishment and solution

2.1 Introduction of TOPSIS

L. Hwang and K. Yoon first proposed TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) in 1981. TOPSIS is a commonly used comprehensive evaluation method, which can make full use of the information of the original data, and its results can accurately reflect the gap between the comparison objects. The following are the steps of TOPSIS algorithm:

Step 1:
Establish the evaluation index system, establish the normalization matrix, standardize the data, remove the influence of dimension.

Suppose there are \( n \) objects to be evaluated, and the matrix of \( m \) evaluation indexes is as follows:

\[
X = \begin{pmatrix}
    x_{11} & x_{12} & \cdots & x_{1m} \\
    x_{21} & x_{22} & \cdots & x_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{n1} & x_{n2} & \cdots & x_{nm}
\end{pmatrix}
\]

Then, the standardized matrix is denoted as \( Z \), and each element of \( Z \):

\[
z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}
\]

Get the matrix \( Z \).

To judge whether there are negative numbers in the \( Z \) matrix, if there are, we need to use another standardization method for \( X \). The formula of standardization is as follows:

\[
\tilde{z}_{ij} = \frac{x_{ij} - \min\{x_{ij}, x_{2j}, \ldots, x_{nj}\}}{\max\{x_{ij}, x_{2j}, \ldots, x_{nj}\} - \min\{x_{ij}, x_{2j}, \ldots, x_{nj}\}}
\]

**Step 2:**

Calculate the proportion of the \( i \)-th sample under the \( j \)-th index, and regard it as the probability used in the calculation of relative entropy.

Suppose there are \( n \) objects to be evaluated, \( m \) evaluation indexes, and the nonnegative matrix obtained by the previous step is as follows:

\[
\tilde{Z} = \begin{pmatrix}
    \tilde{z}_{11} & \tilde{z}_{12} & \cdots & \tilde{z}_{1m} \\
    \tilde{z}_{21} & \tilde{z}_{22} & \cdots & \tilde{z}_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    \tilde{z}_{n1} & \tilde{z}_{n2} & \cdots & \tilde{z}_{nm}
\end{pmatrix}
\]

We calculate the probability matrix \( p \), where the calculation formula of each element \( p_{ij} \) in \( p \) is as follows:

\[
p_{ij} = \frac{\tilde{z}_{ij}}{\sum_{i=1}^{n} \tilde{z}_{ij}}
\]

**Step 3:**

The information entropy of each index is calculated, and the information utility value is calculated, and the entropy weight of each index is obtained by normalization.

For the \( j \)-th index, the calculation formula of its information entropy is as follows:

\[
e_{j} = -\frac{1}{\ln n} \sum_{i=1}^{n} p_{ij} \ln(p_{ij}), j = 1, 2, \ldots, m,
\]
Get the information utility value:
\[ d_j = 1 - e_j, \]

By normalizing the information utility value, we can get the entropy weight of each index:
\[ W_j = \frac{d_j}{\sum_{j=1}^{m} d_j}, \quad j = 1, 2, ..., m. \]

**Step 4:**
The scores were calculated and normalized.
According to the standardized matrix \( Z \) calculated above, define maximum value:
\[ Z^+ = (Z^1_1, Z^2_1, ..., Z^m_1, \max \{z_{i1}, z_{i2}, ..., z_{in}\}, \max \{z_{i1}, z_{i2}, ..., z_{in}\}, ..., \max \{z_{im}, z_{im}, ..., z_{im}\}) \]
Define minimum value:
\[ Z^- = (Z^1_1, Z^2_1, ..., Z^m_1, \min \{z_{i1}, z_{i2}, ..., z_{in}\}, \min \{z_{i1}, z_{i2}, ..., z_{in}\}, ..., \min \{z_{im}, z_{im}, ..., z_{im}\}) \]
Define the distance between the \( i(i = 1, 2, ..., n) \) evaluation object and the maximum value:
\[ D_i^+ = \sqrt{\sum_{j=1}^{m} \omega_j (Z_j^+ - z_j)^2} \]
Define the distance between the \( i(i = 1, 2, ..., n) \) evaluation object and the minimum value:
\[ D_i^- = \sqrt{\sum_{j=1}^{m} \omega_j (Z_j^- - z_j)^2} \]
Then, we can calculate the non normalized score of the \( i(i = 1, 2, ..., n) \) evaluation object:
\[ S_i = \frac{D_i^-}{D_i^+ + D_i^-}, \]
The scores were normalized:
\[ \tilde{S}_i = S_i / \sum_{i=1}^{n} S_i, \]
The \( \tilde{S}_i \) we get is the score of each evaluation object.

### 2.2 Establishment of comprehensive evaluation index system

Higher education depends on institutions of higher learning. The development of higher education is mainly reflected in the relevant aspects of institutions of higher learning. This paper selects eight relevant index (as shown in Figure 1) from the following six aspects to evaluate and analyze the development of higher education.
2.3 Data acquisition and preprocessing

Two developed countries and two developing countries are selected for evaluation and comparison. We choose the United States, Japan, China and Vietnam as the research objects. Based on the eight index selected from the six dimensions, entropy weight method is used to calculate the weight of each index, then TOPSIS method is used to evaluate the higher education system of four countries, and each country is ranked according to the evaluation score.

The original data of the index are obtained from the official data of the statistical departments of various countries. For each index, we have collected relevant data in the past decade. The data of each index collected above within ten years are processed to calculate the average value as the value of a country in a certain index.

2.4 Determination of weight

In order to eliminate the influence of different dimensions on the evaluation results and make the evaluation indexes compare under the same dimension system, it is necessary to standardize the original data. According to the above data, the standardized matrix is obtained

\[
\tilde{Z} = \begin{pmatrix}
0.3412 & 0.7836 & 0.6232 & 0.7042 & 0.5107 & 0.6678 & 0.8799 & 0.5305 \\
0.9328 & 0.6122 & 0.6250 & 0.7043 & 0.5055 & 0.4821 & 0.4735 & 0.5106 \\
0.1048 & 0.0871 & 0.4701 & 0.0821 & 0.4736 & 0.5063 & 0.0398 & 0.3340 \\
0.0493 & 0.0606 & 0 & 0.0370 & 0.5093 & 0.2553 & 0.0002 & 0.5885
\end{pmatrix}
\]

Using entropy weight method, according to the influence degree and importance of each index on higher education, the final weights are as follows:

\[
(0.2025, 0.1706, 0.1308, 0.1879, 0.0002, 0.0224, 0.2767, 0.0089)
\]
2.5 Calculate the score of each evaluation object

According to TOPSIS method, the scores $s$ of four countries are 0.425186191397886, 0.437025637889625, 0.120006434266502 and 0.017781736445986 respectively. That is to say, according to the above eight index to evaluate the higher education of a country, the conclusion is that the higher education of the United States is the healthiest, followed by China, followed by Japan, and Vietnam is the last of the four countries.

3. Evaluation of the models

The weighted TOPSIS comprehensive evaluation model has no strict restrictions on the data distribution and sample size index, which is suitable for both small sample and large-scale system with multi evaluation unit and multi index. On the other hand, TOPSIS model avoids the subjectivity of the data, does not need the objective, the function does not need to pass the test, and can well describe the comprehensive impact of multiple impact indicators. However, TOPSIS model needs the data of each index, so it is difficult to select the corresponding quantitative index. The quantitative selection of some indicators can not fully represent this indicator.

References