

A Node Centrality Evaluation Model for Weighted Social Networks

Peng Wang*

School of Economics and Management, Dalian University, No.10, Xuefu Avenue, Economic & Technical Development Zone, Dalian, Liaoning, The People's Republic of China(PRC)

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Abstract: In this paper, we apply Principal Component Centrality (PCC), a centrality measure for unweighted networks, to weighted social networks, and propose a weighted centrality measure based on tie strength matrix (TSM). Experiment results show that weighted PCC outperforms weighted EVC (EigenVector Centrality) in spreading effectiveness, robustness and tolerance, hence is feasible and effective in weighted social networks.

1. Introduction

Identifying key nodes in the network is an important research component of complex networks, including social networks. For example, in various social networks, it is often necessary to know which are the most active and influential users in order to provide guidance for operators to provide marketing strategies or to provide a useful basis for content distribution of mobile social networks. In social network analysis, “centrality” is used to describe the importance of nodes in the network [1].

Commonly used metrics are centrality, median centrality, near-centrality, and feature vector centering (EVC). Compared with several other central metrics, EVC not only considers the degree of nodes (ie the number of neighbor nodes), but also considers the importance of neighbor nodes, thus becoming a successful method for detecting the most influential nodes in social networks. It is widely used in the social sciences. Since EVC is the main feature vector corresponding to the adjacency matrix as the centrality of the node, the most important nodes are concentrated in one community; and the fact is that these most influential nodes belong to different communities. In view of this, a principal component centrality (PCC) is proposed, which uses the first P eigenvectors of the adjacency matrix to calculate the node centrality, effectively avoiding the defects of EVC. For the defect that the PageRank method is not unique in the non-connected network, the LeaderRank method is proposed. A node that is bidirectionally connected with all nodes is added to the network, so that the whole network is connected and the order is unique. The propagation efficiency and robustness are achieved. There is a clear improvement in terms of fault tolerance. Aiming at the low correlation of degree centrality and the high complexity of the centrality and close centrality in large networks, a semi-local centrality is proposed, which can better identify the influence while reducing the computational complexity. High-powered nodes [2].

The above methods are all directed to an unprivileged network, and only consider whether the nodes are connected, and do not consider the strength of the links between the nodes. Many links in the network do not only indicate presence or absence, but have corresponding weights to record the strength of the link. That is, many networks are weighted networks. In many cases, traditional methods in unprivileged networks can be applied to solve the problem of weighted networks. Therefore, extending the method of unprivileged networks to weighted networks is one of the important issues in network research. The EVC in the unprivileged network can be extended to the weighted network, and the weighted network EVC thus obtained is used for ranking the search results of the citation network; however, the PCC is a better tool than the EVC in terms of the centrality of the computing node, and there is currently no expanded to a weighted network.

This paper first proposes to apply the PCC of the metric node centrality in the weightless network to the weighted network, and propose a weighted principal component centrality metric (weighted PCC) based on the link strength matrix. Experimental results show that weighted PCC is superior to weighted EVC in terms of propagation efficiency, robustness and fault tolerance. Therefore, weighted PCC is feasible and effective in weighted social networks. The most important contribution of this paper is to extend the PCC of the unprivileged network to the weighted social network, thus enriching the study of extending the classical method of the unprivileged network to the weighted network.

2. Unweighted Network Node Centrality Calculation Method

Commonly used metrics are centrality, median centrality, near-centrality, and feature vector centering (EVC). Compared with several other central metrics, EVC not only considers the degree of nodes, but also considers the importance of neighbor nodes, so it is a successful method to detect the most influential nodes in social networks. The EVC of a node in an unprivileged network is defined as being proportional to the sum of the EVCs of the neighbors of the node [3].

Suppose $A (A_{ij})$ is the adjacency matrix of the network graph $G (V, E)$. Assume that the definition of EVC in an unprivileged network is assumed. By analyzing the convergence of the EVC algorithm, if $1/\lambda$ is the largest eigenvalue of the modulus of the matrix A , then the eigenvector centrality x should be the eigenvector corresponding to $1/\lambda$, also known as the main eigenvector. EVC has its shortcomings when portraying the centrality of unauthorised network nodes: the most important nodes are concentrated in a small area, that is, EVC regards the most important nodes as a small community, and the fact is that the most important Some nodes may belong to different communities; in addition, the EVC values of most nodes are meaningless 0, which can not fully meet the needs of applications such as sorting.

By mapping the network graph to a multiple graph, the unweighted network node centrality metric can be extended to the weighted network; then, the EVC of the node in the weighted network is the main eigenvector of the current weighted adjacency matrix. The weighted network EVC thus obtained can be used for ranking the search results of the citation network. Since the weighted network can be regarded as a multiple map mapped by an unprivileged network, EVC also has the same problem as an unprivileged network when characterizing the centrality of the weighted network node.

3. Weighted Network Node Centrality Calculation Model

3.1 Link Strength Matrix

The traditional central computing method fails to consider the time dynamics of the connection between nodes, and the link strength can overcome this defect by specifically describing the

probability that the connection is available. Therefore, this article uses link strength to characterize the weight of links between nodes.

Link strength is a property of quantitatively portraying links between nodes. The strength of a relationship depends on four factors: contact frequency, relationship duration, contact duration, and number of interactions. Based on this, the researchers propose seven factors and use different factors to characterize the link strength according to different research needs: frequency, intimacy, longevity, mutuality, proximity, multiple social backgrounds, and trust.

This paper selects two factors, frequency and intimacy, to characterize the link strength. The more frequent the interaction occurs on the link, the more intimate (ie, the longer the connection duration), the stronger the link strength. The calculation of these two factors is based on an evidence-based strategy that measures the trust of a node or system with evidence by the ratio of the number of supporting evidence to the number of rebuttal evidence.

(1) Frequency factor: Depending on the frequency at which a node i meets other nodes j , it is calculated by the number of encounters between nodes.

(2) Intimacy factor: that is, the connection duration depends on the length of time that a node i is connected to other nodes

3.2 Weighted Network Node Centrality Calculation Model

Let A be the adjacency matrix of the unprivileged network $G(V, E)$, where D_i is the node set and E is the undirected edge set. The eigenvalues of A are arranged in the order of decreasing modulus.

$D_1 = \int \xi d_1(p_1) dt + x_1$, the corresponding characteristic vector is $D_2 = \int_{t_0}^T d_2(p_2) dt + x_2$. The PCC in the unprivileged network is defined as the Euclidean distance between the node and the origin in the P -dimensional feature space, wherein the base vector of the P -dimensional feature space is the first P feature vectors $d_i(p_i) = a_i - k_i(1 - \theta(t)) - b_i p_i$, and $a_1 > a_2 > 0$, then $b_1 > b_2 > 0$ is a matrix of the first P eigenvectors, $\mu_i = \bar{x}_i$ is composed of the first P eigenvalues vector.

4. Conclusions

Determining key nodes in the network is important in social network research. In this paper, the PCC method for measuring node centrality in an unprivileged network is extended, and a central computing model (weighted PCC) suitable for weighted social networks is proposed based on the link strength matrix. Experimental results show that weighted PCC is superior to weighted EVC in the comparison of propagation efficiency, robustness and fault tolerance. This result shows that applying PCC to measure and sort the importance of weighting social network nodes can effectively promote the diffusion of information, which is valuable in practical applications, such as accelerating information dissemination, controlling the spread of public opinion, accelerating the content distribution of mobile social networks, etc. At the same time, because weighted PCC has better robustness against fake fans, it has better fault tolerance for noise data, making it feasible and effective in the actual weighted social network platform.

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