An Empirical Study of the Markowitz Portfolio Model on Nasdaq Stocks

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Abstract: The Markowitz Model has been proposed for 70 years, however, due to many constraints in the modern investment process, it is not possible to apply the Markowitz Model directly to modern investments. The aim of this paper is to address the deficiency of the Markowitz Model whose reference value has been reduced due to lacking consideration of various constraints in the modern investment process and to address the problem of the Markowitz Model being out of date in the modern investment process research. This paper uses the price data of 10 stocks in the NASDAQ for the latest 20 years and applies the Markowitz Model theory to obtain the optimal portfolio under the common constraints of modern investment, thus applying the Markowitz Model to modern investment situations. The result shows that the Markowitz Model can still arrive at a reliable optimal investment portfolio after considering various constraints. Markowitz's investment theory still has reliable guiding value in the modern investment process.

1. Introduction

The securities investment portfolio mainly discusses the relationship between the risk and the return of the portfolio composed of various securities as a whole and how investors can reasonably allocate their investment amount in the portfolio [1]. The investment portfolio is not a simple random combination of securities types. It reflects the investor's will and the constraints imposed by the investor, that is, the investor's trade-off of investment returns, the allocation of investment proportions, and investment risk preferences [2].

 Stocks are a common type of securities. Investing in stocks can bring huge returns, but returns are often accompanied by corresponding risks. There is still a huge risk in investing in stocks. Investors should not only be limited to the return forecast of stock investment but also take into account the economic losses caused by the occurrence of risks [3]. If the relationship between the two cannot be balanced, investors may face huge losses. How to balance the relationship between risk and return has become a problem that investors pay close attention to.

Markowitz Portfolio Theory is the beginning of portfolio theory and lays the foundation for subsequent portfolio decisions. Although there are many defects in the model set, it quantifies the two key indicators of investment income and risk and proposes a mathematical model, which enables investors to have the earliest decision-making basis. The core of the Markowitz model is to minimize risk on the basis of a given rate of return—that is, to reduce unsystematic risk as much as possible by diversifying investment, which also means that investors try to choose as much as possible in the selection of investment portfolios less correlated assets. Theoretically, on the basis of rational allocation of investment assets, investors can reduce the unsystematic risk of the investment portfolio to zero, so as to obtain the expected rate of return that only bears the market risk [4].

Markowitz's portfolio theory has played an important guiding role in the theoretical investment process since its inception, but there are many defects in practical application. The main reason is that the implementation of the model is based on a set of strict preset conditions, which are almost impossible to hold in reality. We still need to note that the Markowitz model does not take into account some practical restrictions in the real securities market [5], such as the regulation of transaction costs, the minimum transaction size limit, and the prohibition of short-selling and short-selling in the Chinese
securities market. If these constraints faced in the actual transaction process are not taken into account in the model, the usefulness of the model will be greatly reduced [6].

Recently, there have been many variations and additions to the Markowitz model. Some scholars have also tinkered with the Markowitz Model, for example, by proposing "Markowitz with regret", making regret as an additional decision criterion [7]. Some scholars have used the Markowitz model to solve the long-term investment problem through a cointegration strategy of pair trading [8]. Some scholars have added the dimension of Time to Mean-Variance, forming the Mean-variance-time model [9]. Some scholars have developed measures of risk appetite for non-Markowitz factors to classify investors' risk appetite levels [10].

Investors are exposed to systematic and unsystematic risks. Using a diversified strategy, investment managers can reduce the unsystematic risk of their portfolios and buy safer, higher-returning portfolios for their capital providers. More than 70 years after its inception, the Markowitz mean-variance model remains unchallenged in the investment. However, in recent years, the US government has imposed new restrictions on portfolios, while funders have made more additional conditions on portfolios and most of the articles applying the Markowitz model to the analysis of portfolios have become outdated.

This paper analyses the price fluctuation of 10 stocks in the NASDAQ over the last 20 years and applies the Markowitz model under a number of realistic constraints to arrive at the optimal portfolio, filling a gap left by the outdated study of the Markowitz model.

The remainder of the paper is organized as follows: Section 2 describes the price fluctuation and correlations of stocks; Section 3 introduces the classical Markowitz Model and related constraints; Section 4 presents the portfolios and investment evaluation indicators under different constraints; The last section presents our conclusions.

2. Data

This paper searched for historical daily total return data of the recent 20 years for ten stocks, which belong in groups to three-four different sectors, one (S&P 500) equity index (a total of eleven risky assets), and a proxy for risk-free rate (1-month Fed Funds rate). In order to reduce the non-Gaussian effects, the researcher aggregated the daily data to the monthly observations and based on those monthly observations, calculated all proper optimization inputs for the full Markowitz Model. Using these optimization inputs for MM, the researcher will find the regions of permissible portfolios (efficient frontier, minimal risk portfolio, optimal portfolio, and minimal return portfolios frontier) for the five cases of the additional constraints.

2.1 Price fluctuation of the ten stocks

Qualcomm is the world's leading wireless technology innovator and a driving force in the development, commercialization, and scale-up of 5G.

Founded in 1985, Qualcomm employs approximately 37,000 people worldwide, had fiscal 2020 revenues of $23,531 million, and had invested more than $61 billion in research and development by the end of 2019.

Qualcomm's businesses include technology-leading 3G and 4G chipsets, system software and development tools and products, technology licensing, the BREW application development platform, QChat and BREWChat VoIP solution technologies, QPoint location solutions, Eudora email software, comprehensive wireless solutions including two-way data communications systems, wireless consultancy, and network management services, MediaFLO systems and GSM1x technology.

Qualcomm has been named to the Fortune 500 for more than 10 consecutive years and was ranked No. 1 on Fortune's 2019 list of "Companies Changing the World" for its significant contributions to the development of wireless technology and its drive for 5G.
Akamai is the world's largest and most trusted cloud delivery platform making it easier for its customers to provide the best and most secure digital experiences. Akamai aims to eliminate Internet bottlenecks and improve the Internet user experience, protect end-user access to Web sites and on-premises applications, and provide comprehensive cloud security services that accelerate HD streaming worldwide.

Founded in 1998, Akamai employs over 8,000 people worldwide and had 2020 revenues of $3.2 billion, up 11% year-over-year.

Akamai has deployed the most pervasive, highly-distributed content delivery network (CDN) in more than 135 countries and over 1,400 networks worldwide, with approximately 325,000 servers.

For the second year in a row, Akamai was ranked #1 on the Boston Business Journal's 2019 list of "Massachusetts' Largest Cybersecurity Companies".

Oracle is the world's largest provider of information management software and services, with virtually every industry in the world using Oracle technology and 98 of the Fortune 100 companies using Oracle technology. Oracle is the world's leading provider of information management software and the second-largest independent software company in the world.
Founded in 1977, Oracle employs approximately 137,000 employees (2019) and had 2020 revenues of $39,506 million.

Oracle's main businesses include the server business, represented by database servers and application servers, and application software, represented by enterprise resource planning software, customer relationship management software, and human resource management software.

In 2013, Oracle has overtaken IBM to become the second-largest software company in the world after Microsoft.

Figure 3. Oracle's Price fluctuations in the last twenty years

Microsoft is a multinational technology company, the world's largest provider of computer software, and a pioneer in the development of the world's PC software with a focus on developing, manufacturing, licensing, and providing a wide range of computer software services.

Founded in 1975, Microsoft employs nearly 190,000 people worldwide and had annual revenues of $125,843 million in 2020.

Its main businesses include operating systems, office software, tablets, gaming consoles, cloud services, and more, with its best-known and best-selling products being the Windows operating system and the Office family of software.


Figure 4. Microsoft's Price fluctuations in the last twenty years
Chevron Corporation is one of the world's largest integrated energy companies. With more than a century of leadership in product innovation and customer value creation, the company provides products and services under the Chevron, Texaco, and Caltex brands to customers in more than 100 countries around the world.

Founded in 1879, Chevron employs about 48,600 people worldwide (2019) and had annual revenues of $146.5 billion in 2020.

With a global presence in more than 180 countries, Chevron Corporation is present in all aspects of the oil and gas industry: exploration, production, refining, marketing, transportation, petrochemicals, power generation, and more.

In June 2021, Chevron was ranked 27th on the 2021 Fortune 500 list.

ExxonMobil is one of the world's largest publicly traded energy providers and chemical manufacturers, develops and applies next-generation technologies to help safely and responsibly meet the world's growing needs for energy and high-quality chemical products.

Founded in 1882, ExxonMobil employs about 74,900 people worldwide and had annual revenues of $264.9 billion in 2020.

ExxonMobil is an industry leader in many aspects of energy and petrochemicals, with oil and gas exploration operations in approximately 200 countries and territories around the world through its affiliated companies. It is also one of the world's largest refiners, with a refining capacity of 6.4 million barrels per day at 45 refineries in 25 countries; more than 37,000 gas stations and 1 million industrial and wholesale customers worldwide; and approximately 28 million tons of petrochemicals sold annually in more than 150 countries.

In June 2021, ExxonMobil was ranked No. 10 on the 2021 Fortune 500 list.
Imperial Oil is an industry leader in applying technology and innovation to responsibly develop Canada's energy resources. As an integrated energy producer, it explores for, produces, refine, and markets products that empower modern living.

Imperial Oil Limited explores for, produces, and sells crude oil and natural gas in Canada. It operates through three segments: Upstream, Downstream, and Chemical.

Imperial is distinguished for its long-term commitment to research and technology. It is one of the select few energy companies in Canada with dedicated research facilities. They produce and provide quality petrochemical products and services with a commitment to the principles of sustainability.

Coca-Cola is the world's largest beverage manufacturer and the world's largest distributor of juice drinks. It has a 48% global market share. Every day, 1.7 billion people around the world drink Coca-Cola products, and approximately 19,400 bottles are sold every second.

Founded in 1886, it employs about 62,600 people worldwide and had annual revenues of $37.2 billion in 2020.
Coca-Cola has a grand presence in several beverage programs. Coca-Cola has 160 beverage brands in 200 countries, including soft drinks, sports drinks, dairy drinks, juices, teas, and coffees. Sprite is its fastest-growing product.

In October 2016, The Coca-Cola Company was ranked third in the 100 most valuable brands in the world in 2016. In June 2021 and ranked 93rd on the Fortune 500 US list for 2021.

PepsiCo is one of the world's leading food and beverage companies, serving more than 200 countries and territories worldwide. The company is made up of seven divisions: PepsiCo Beverages North America; Frito-Lay North America; Quaker Foods North America; Latin America; Europe; Africa, Middle East and South Asia; and Asia Pacific, Australia/New Zealand, and China. Each of these divisions has its own unique history and way of doing business.

Founded in 1965, it employs about 267,000 people worldwide and had annual revenues of $67.2 billion in 2020.

PepsiCo has also found success in other beverage products, including Mountain Dew beverages. On 10 August 2020, PepsiCo was ranked 160th on the Fortune 500 list for 2020.

McDonald's is a major global restaurant chain with more than 32,000 locations in 121 countries and territories worldwide.

Founded in 1955, it employs about 420,000 people worldwide (2015) and had annual revenues of $21.1 billion in 2020.
McDonald's sells hamburgers, as well as fast food items such as fries, fried chicken, sodas, ice cream, salads, and fruit, and also controls other restaurant brands such as Aroma Cafe, Boston Market, Chipotle, Donatos Pizza, and Pret a Manger. In May 2021, McDonald's will launch a new restaurant.

In May 2021, McDonald's was ranked 201st on the 2021 Forbes Global 2000 list.

![McDonald's Stock Price Fluctuations](image)

Figure 10. McDonald's Price fluctuations in the last twenty year

### 2.2 Correlations of the ten stocks

The correlations of 10 stocks from several sectors are shown below (Table 1). As the table indicates, the correlations of the returns of these 10 stocks are low and satisfy the requirements for risk diversification.

<table>
<thead>
<tr>
<th>Correlations</th>
<th>SPX</th>
<th>QCOM</th>
<th>AKA</th>
<th>ORC</th>
<th>MSFT</th>
<th>CVX</th>
<th>XOM</th>
<th>IMO</th>
<th>KO</th>
<th>PEP</th>
<th>MCD</th>
</tr>
</thead>
<tbody>
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<td>SPX</td>
<td>1.00</td>
<td>0.557</td>
<td>0.389</td>
<td>0.545</td>
<td>0.638</td>
<td>0.612</td>
<td>0.568</td>
<td>0.522</td>
<td>0.491</td>
<td>0.522</td>
<td>0.537</td>
</tr>
<tr>
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<td>0.284</td>
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<td>0.233</td>
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<td>0.272</td>
<td>0.196</td>
<td>0.263</td>
<td>0.261</td>
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<td>0.278</td>
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<td>0.242</td>
<td>0.256</td>
<td>0.121</td>
<td>0.068</td>
<td>0.126</td>
<td>0.084</td>
<td>0.101</td>
<td>0.290</td>
</tr>
<tr>
<td>ORCL</td>
<td>0.545</td>
<td>0.284</td>
<td>0.242</td>
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<td>0.474</td>
<td>0.263</td>
<td>0.301</td>
<td>0.233</td>
<td>0.067</td>
<td>0.205</td>
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<tr>
<td>MSFT</td>
<td>0.638</td>
<td>0.375</td>
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<td>0.304</td>
<td>0.249</td>
<td>0.279</td>
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<td>0.356</td>
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<tr>
<td>CVX</td>
<td>0.612</td>
<td>0.233</td>
<td>0.121</td>
<td>0.263</td>
<td>0.338</td>
<td>1.000</td>
<td>0.829</td>
<td>0.734</td>
<td>0.401</td>
<td>0.271</td>
<td>0.393</td>
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<td>XOM</td>
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<td>0.234</td>
<td>0.068</td>
<td>0.301</td>
<td>0.304</td>
<td>0.829</td>
<td>1.000</td>
<td>0.696</td>
<td>0.337</td>
<td>0.239</td>
<td>0.340</td>
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<tr>
<td>IMO</td>
<td>0.522</td>
<td>0.272</td>
<td>0.126</td>
<td>0.233</td>
<td>0.249</td>
<td>0.734</td>
<td>0.696</td>
<td>1.000</td>
<td>0.296</td>
<td>0.178</td>
<td>0.267</td>
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<tr>
<td>KO</td>
<td>0.491</td>
<td>0.196</td>
<td>0.084</td>
<td>0.067</td>
<td>0.279</td>
<td>0.401</td>
<td>0.337</td>
<td>0.296</td>
<td>1.000</td>
<td>0.579</td>
<td>0.499</td>
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<tr>
<td>PEP</td>
<td>0.522</td>
<td>0.263</td>
<td>0.101</td>
<td>0.205</td>
<td>0.334</td>
<td>0.271</td>
<td>0.239</td>
<td>0.178</td>
<td>0.579</td>
<td>1.000</td>
<td>0.469</td>
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<tr>
<td>MCD</td>
<td>0.537</td>
<td>0.261</td>
<td>0.290</td>
<td>0.137</td>
<td>0.356</td>
<td>0.393</td>
<td>0.340</td>
<td>0.267</td>
<td>0.499</td>
<td>0.469</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 1. Correlations of the ten stocks
3. METHODS

Based on the recent 20 years of stock market prices, use the Markowitz model to explore the permissible portfolios region and the optimal portfolio of different investments under different constraints, which is useful for investors in decision analysis.

3.1 Markowitz Model

Suppose there are n risky assets in the market and the benefits of the assets are r1, r2, … rn, and the investor's allocation to each risky asset is ω1, ω2, … ωn.

the benefits of the portfolio are:

\[ r_p = \sum_{i=1}^{n} \omega_i r_i \]  

(1)

\[ \sum_{i=1}^{n} \omega_i = 1 \]  

(2)

Thus, the expected return rate of the portfolio and variance are:

\[ E(r_p) = \sum_{i=1}^{n} \omega_i E(r_i) \]  

(3)

\[ \text{Var}(r_p) = \sum_{i=1}^{n} \omega_i^2 \text{Var}(r_i) + \sum_{i \neq j} \omega_i \omega_j \text{Cov}(r_i, r_j) \]  

(4)

3.2 Constraints

3.2.1 Constraint 1

\[ \sum_{i=1}^{11} |w_i| \leq 2 \]  

(5)

This first optimization constraint is designed to simulate Regulation T by FINRA, which allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity. The terms on which firms can extend credit for securities transactions are governed by federal regulation and by the rules of FINRA and the securities exchanges. Some securities cannot be purchased on margin, which means they must be purchased in a cash account, and the customer must deposit 100 percent of the purchase price. In general, under Federal Reserve Board Regulation T, firms can lend a customer up to 50 percent of the total purchase price of margin security for new, or initial, purchases. The rules of FINRA and the exchanges supplement the requirements of Regulation T by placing "maintenance" margin requirements on customer accounts.

Under these rules, as a general matter, the customer's equity in the account must not fall below 25 percent of the current market value of the securities in the account. Otherwise, the customer may be required to deposit more funds or securities to maintain equity at the 25 percent level (referred to as a margin call). Failure to do so may cause the firm to liquidate the securities in the customer's account in order to bring the account's equity back up to the required level.

3.2.2 Constraint 2

\[ |w_i| \leq 1, \text{for } \forall i \]  

(6)

This second optimization constraint is designed to simulate some arbitrary “box” constraints on weights, which may be provided by the client.
3.2.3 Constraint 3

\[ w_i \geq 0, \text{ for } \forall i \]  

(7)

This additional optimization constraint is designed to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions.

3.2.4 Constraint 4

\[ w_i = 0 \]

(8)

we would like to see if the inclusion of the broad index into our portfolio has a positive or negative effect, for we would like to consider an additional optimization constraint, which means that the S&P 500 equity index equals zero.

4. Result ANALYSIS

Using the formula of the Markowitz Model and the Solver function of Excel, we can get the portfolios with the smallest variance and the largest Sharpe ratio under different constraints.

4.1 Constraint 1

We obtained two portfolios of ten stocks under Restriction 1 (Table 2), and the returns, standard deviations, and Sharpe ratios of these two portfolios (Table 3). These two portfolios achieved the smallest variance and the largest Sharpe ratio under Restriction 1, respectively. In the minimal variance point, the return of the portfolio is 7.232%, the standard deviation is 12.279% and the Sharpe ratio is 0.589. In the maximal shape ratio point, the return of the portfolio is 14.587%, the standard deviation is 16.112% and the Sharpe ratio is 0.905.

<table>
<thead>
<tr>
<th>Weights</th>
<th>SPX</th>
<th>QCOM</th>
<th>AKA</th>
<th>ORCL</th>
<th>MSFT</th>
<th>CVX</th>
<th>XOM</th>
<th>IMO</th>
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<th>PEP</th>
<th>MC</th>
<th>D</th>
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<tbody>
<tr>
<td>MinVar</td>
<td>0.309</td>
<td>-</td>
<td>-</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
<td>0.195</td>
<td>-</td>
<td>0.20</td>
<td>0.30</td>
<td>0.08</td>
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<tr>
<td></td>
<td>5</td>
<td>0.021</td>
<td>0.010</td>
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<td>0.087</td>
<td>6</td>
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<td>96</td>
<td>52</td>
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<tr>
<td>MaxSha</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.060</td>
<td>0.062</td>
<td>0.14</td>
<td>0.208</td>
<td>0.057</td>
<td>-</td>
<td>0.115</td>
<td>0.05</td>
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<tr>
<td>rpe</td>
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<td>1</td>
<td>12</td>
<td>98</td>
<td>55</td>
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</table>

Table 3. Related indicators for the portfolios under constraint 1

<table>
<thead>
<tr>
<th></th>
<th>Return</th>
<th>Standard deviation</th>
<th>Sharpe ratio</th>
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</thead>
<tbody>
<tr>
<td>MinVar</td>
<td>7.232%</td>
<td>12.279%</td>
<td>0.589</td>
</tr>
<tr>
<td>MaxSharpe</td>
<td>14.587%</td>
<td>16.112%</td>
<td>0.905</td>
</tr>
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</table>

4.2 Constraint 2

We obtained two portfolios of ten stocks under Restriction 2 (Table 4), and the returns, standard deviations, and Sharpe ratios of these two portfolios (Table 5). These two portfolios achieved the smallest variance and the largest Sharpe ratio under Restriction 2, respectively. In the minimal variance point, the return of the portfolio is 7.232%, the standard deviation is 12.279% and the Sharpe ratio is 0.589. In the maximal shape ratio point, the return of the portfolio is 17.900%, the standard deviation is 16.112% and the Sharpe ratio is 0.905.
Table 4. Weights of each stock under constraint 2

<table>
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<tr>
<th>Weights</th>
<th>SPX</th>
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<th>AKA</th>
<th>ORC</th>
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<th>IMO</th>
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<th>PEP</th>
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<td>MinVar</td>
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<td>0.05</td>
<td>0.003</td>
<td>0.087</td>
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<tr>
<td></td>
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<td>5</td>
<td>33</td>
<td>8</td>
<td>6</td>
<td>6</td>
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<tr>
<td>MaxSharpe</td>
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Table 5. Related indicators for the portfolios under constraint 2

<table>
<thead>
<tr>
<th></th>
<th>Return</th>
<th>Standard deviation</th>
<th>Sharpe ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinVar</td>
<td>7.232%</td>
<td>12.279%</td>
<td>0.589</td>
</tr>
<tr>
<td>MaxSharpe</td>
<td>17.900%</td>
<td>19.189%</td>
<td>0.933</td>
</tr>
</tbody>
</table>

4.3 Constraint 3

We obtained two portfolios of ten stocks under Restriction 3 (Table 6), and the returns, standard deviations, and Sharpe ratios of these two portfolios. (Table 7).

Table 6. Weights of each stock under constraint 3

<table>
<thead>
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<th>Weights</th>
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<th>ORC</th>
<th>MSF</th>
<th>CVX</th>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>6</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>MaxSharpe</td>
<td>0.000</td>
<td>0.021</td>
<td>0.053</td>
<td>0.083</td>
<td>0.148</td>
<td>0.000</td>
<td>0.000</td>
<td>0.053</td>
<td>0.000</td>
<td>0.197</td>
<td>0.443</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>8</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Related indicators for the portfolios under constraint 3

<table>
<thead>
<tr>
<th></th>
<th>Return</th>
<th>Standard deviation</th>
<th>Sharpe ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinVar</td>
<td>8.025%</td>
<td>12.398%</td>
<td>0.647</td>
</tr>
<tr>
<td>MaxSharpe</td>
<td>12.759%</td>
<td>15.065%</td>
<td>0.847</td>
</tr>
</tbody>
</table>

These two portfolios achieved the smallest variance and the largest Sharpe ratio under Restriction 3, respectively. In the minimal variance point, the return of the portfolio is 8.025%, the standard deviation is 12.398% and the Sharpe ratio is 0.647. In the maximal shape ratio point, the return of the portfolio is 12.759%, the standard deviation is 15.065% and the Sharpe ratio is 0.847.

4.4 Constraint 4

We obtained two portfolios of ten stocks under Restriction 4 (Table 8), and the returns, standard deviations, and Sharpe ratios of these two portfolios. (Table 9) These two portfolios achieved the smallest variance and the largest Sharpe ratio under Restriction 4, respectively. In the minimal variance point, the return of the portfolio is 8.162%, the standard deviation is 12.457% and the Sharpe ratio is 0.655. In the maximal Sharpe ratio point, the return of the portfolio is 14.383%, the standard deviation is 16.537% and the Sharpe ratio is 0.870.
Table 8. Weights of each stock under constraint 4

<table>
<thead>
<tr>
<th>Weights</th>
<th>SPX</th>
<th>QCO</th>
<th>AKA</th>
<th>ORC</th>
<th>MSF</th>
<th>CVX</th>
<th>XOM</th>
<th>IMO</th>
<th>KO</th>
<th>PEP</th>
<th>MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinVar</td>
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<td>0.008</td>
<td>0.000</td>
<td>0.09</td>
<td>0.03</td>
<td>0.217</td>
<td>0.24</td>
<td>0.36</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>18</td>
<td>72</td>
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<td>1</td>
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<td>9</td>
<td>81</td>
<td>60</td>
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<tr>
<td>MaxSharpe</td>
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<td>0.024</td>
<td>0.048</td>
<td>0.11</td>
<td>0.16</td>
<td>0.130</td>
<td>0.326</td>
<td>0.133</td>
<td>0.008</td>
<td>0.21</td>
<td>0.50</td>
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<tr>
<td></td>
<td>0.00</td>
<td>2</td>
<td>57</td>
<td>17</td>
<td>2</td>
<td>0.09</td>
<td>6</td>
<td>9</td>
<td>69</td>
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</tr>
</tbody>
</table>

Table 9. Related indicators for the portfolios under constraint 4

<table>
<thead>
<tr>
<th></th>
<th>Return</th>
<th>Standard deviation</th>
<th>Sharpe ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinVar</td>
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<td>12.457%</td>
<td>0.655</td>
</tr>
<tr>
<td>MaxSharpe</td>
<td>14.383%</td>
<td>16.537%</td>
<td>0.870</td>
</tr>
</tbody>
</table>

5. Conclusion

We have used the Markowitz Model to demonstrate the modern investment process, under a variety of reality-based constraints. We have used data from 10 NASDAQ stocks over the last 20 years and found the minimum variance and maximum Sharpe ratio points of their portfolios under specific constraints and derived portfolio evaluation metrics.

Our results demonstrate that the Markowitz Model is still valid for portfolio selection and evaluation when realistic constraints are taken into consideration, which goes some way to dispelling concerns about the obsolescence of the Markowitz Model in the modern investment process and can help investment managers make more rational decisions.

There are still some limitations to this study, such as the number of stocks selected is not large enough and the constraints may not be customized to reflect the specific constraints faced by investment managers. By combining new features of the modern investment process with classical investment theory, we can continue to develop classical theories and continue to allow them to play a guiding role in investment.

References


