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Advances and Application Prospects of Turbulence Models

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Abstract: Turbulence is a common flow phenomenon in nature and engineering, characterized by irregularity and strong randomness. Turbulence calculation is critical in many engineering designs and applications. To explain the flow pattern of turbulence, scientists have created various models to simulate it. At present, the prediction of turbulence transition and the improvement of large eddy simulation are still not comprehensive, and there is a lack of systematic summary and reasonable research routes. Therefore, this article introduces the simulation ideas and basic control equations of three methods: Reynolds averaged, direct numerical simulation, and large eddy simulation. The development background, research status, and specific applications of large eddy simulation technology are emphasized. The advantages, disadvantages, and applicability of large eddy simulation are analyzed in depth. This article aims to review the development of turbulence models and summarize the current main turbulence models, providing a reference for related research and of great value for future research on large eddy simulation.

1. Introduction

Turbulence, as a ubiquitous and unstable flow field, is a common physical characteristic in irregular flows. After more than a century of research, although there has been a lot of understanding of its flow phenomenon, there is still no accurate description of its flow phenomenon. At present, research on turbulence is still in its early stages, observing turbulence phenomena and attempting to improve turbulence images [1].

In the development of turbulence, different but complementary methods have been developed to simulate turbulent flow. Reynolds Average Navier Stokes is the averaging of Navier Stokes equations, which transforms no steady turbulence problems into a stationary problem for research. However, due to the averaging of time, some equations may be missing, making it more difficult to develop Reynolds stress models. Direct numerical simulation [2] is a method that requires solving all turbulence spatial lengths and time scales and requires a very fine three-dimensional mesh to solve them. This is the most accurate method for simulating turbulence, but it also requires a large time step to provide average statistical data, resulting in high computational costs for direct numerical simulation (DNS) Currently, it is only used in a few fields. Large eddy simulation (LES), as a moderate method, filters the Navier-Stokes equations spatially and only calculates large-scale vortices, while proposing models for small-scale vortices in the inertial region and below. However, it still requires a very large amount of computation, so it was not applied in complex turbulence engineering in the early stages. In recent years, with the improvement of computer computing power, LES has been valued developed, and applied [3].

At present, with the development of computer technology, the accuracy and usability of all turbulence simulations are constantly improving. In this article, the basic principles of the Reynolds averaged method, direct simulation method, and large eddy simulation method are summarized, and typical problems in large eddy simulation are summarized. This study aims to elucidate the mechanism of large eddy simulation and provide prospects for the development of LES.

This article summarizes the three main methods of turbulence modelling and provides a profound explanation of the underlying mechanisms of large eddy simulation, which is of great value for the development of turbulence simulation.

2. Turbulence Simulation Theory

2.1. Reynolds-averaged Navier-Stokes Equation

The main idea of Reynolds averaged turbulence is to decompose the flow field variables into mean and disturbance values, and only consider the mean. The following is a brief description of the time-averaged equations in the continuity equation, momentum equation, and energy equation.

2.1.1. Time-averaged Equation of Continuity Equation

Substituting $u_i = \overline{u}_i + u_i'$into the continuity equation to obtain:

$$\frac{\partial u_j}{\partial x_j} = \frac{\overline{\partial\left(\overline{u_j}+u_j'\right)}}{\partial x_j} = \frac{\partial \overline{u_j}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_j} = \frac{\partial \overline{u_j'}}{\partial x_j} \tag{1}$$

where u_i is the directional velocity component of x_i, p, ρ, and v represent pressure, density, and viscosity coefficient of motion, respectively.

The continuity equation for the time-averaged incompressible flow can be obtained:

$$\frac{\partial \overline{u_j}}{\partial x_j} = 0 \tag{2}$$

2.1.2. The Time-averaged Equation of Momentum Equation

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_j u_i)}{\partial x_j} = f_i - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_{ji}}{\partial x_j} \tag{3}$$

For incompressible fluid flow, typically ρ = Const, therefore $\rho' = 0$, and $\rho = \overline{\rho}$. If ignore the pulsation of the mass force term, there is $f = \overline{f}$. The momentum equation for obtaining time-averaged motion is:

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j}\frac{\partial \overline{u_i}}{\partial x_j} = \overline{f_i} - \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\overline{\tau_{ji}} - \rho\overline{u_j' u_i'}\right) \tag{4}$$

Compared to the momentum equation before calculating the time average, there is an additional term $-\frac{\partial\left(u_j' u_i'\right)}{\partial x_j}$, This term is caused by the nonlinearity of the term flow in the momentum equation, representing the influence of pulsating velocity on the mean flow. Therefore, momentum exchange occurs between pulsating motion and mean motion, causing the average velocity distribution of turbulence to be different from that of laminar flow under the same conditions.

In Formula 4,$-\frac{\partial\left(\overline{u_j'u_i'}\right)}{\partial x_j}$ has the same dimension as viscous stress τ_{ji}, therefore $\tau_{t,ij}=-\rho\overline{u_j'u_i'}$ is called Reynolds stress.

Reynolds stress is a second-order symmetric tensor composed of six independent components:

$$\tau_t=\begin{pmatrix}-\rho\overline{u'u'} & -\rho\overline{u'v'} & -\rho\overline{u'w'}\\ -\rho\overline{v'u'} & -\rho\overline{v'v'} & -\rho\overline{v'w'}\\ -\rho\overline{w'u'} & -\rho\overline{w'v'} & -\rho\overline{w'w'}\end{pmatrix} \tag{5}$$

Subtracting the momentum equation from the time-averaged equation yields the momentum equation for pulsating motion:

$$\frac{\partial u_i'}{\partial t}+u_j'\frac{\partial\overline{u_i}}{\partial x_j}+\overline{u_j}\frac{\partial u_i'}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p'}{\partial x_i}+\frac{1}{\rho}\frac{ji}{\partial x_j}+\frac{\partial\left(\overline{u_j'u_i'}-u_j'u_i'\right)}{2m} \tag{6}$$

$u_j'\frac{\partial\overline{u_i}}{\partial x_j}$ represents the spatial variation of the average velocity of migration with pulsating motion. $\frac{\partial u_i'}{\partial x_j}$ represents the spatial variation of the pulsating velocity due to the migration of average motion.

2.1.3. Time-averaged Equation of Energy Equation

$$\rho c_p\left(\frac{\partial T}{\partial t}+\frac{\partial(u_jT)}{\partial x_j}\right)=\frac{\partial}{\partial x_j}\left(-q_j\right)+S \tag{7}$$

where c_p is the specific heat capacity, S is the internal heat source of the fluid and the part where the mechanical energy of the fluid is converted into thermal energy due to viscosity, and T is the temperature.

Calculate the time-averaged of Formula 7 to obtain the energy equation for constant physical properties, incompressible fluids, and time-averaged motion:

$$\rho c_p\left(\frac{\partial\overline{T}}{\partial t}+\overline{u_j}\frac{\partial\overline{T}}{\partial x_j}\right)=\frac{\partial}{\partial x_j}\left(-\overline{q_j}-\rho c_p\overline{u_j'T'}\right)+S\left(\overline{T}\right) \tag{8}$$

From Formula 8, it can be concluded that the incompressible Reynolds time averaged equation system is:

$$\frac{\partial\overline{u_j}}{\partial x_j}=0 \tag{9}$$

$$\frac{\partial\overline{u_i}}{\partial t}+\frac{\partial\overline{u_i}}{\partial x_j}=\overline{f_i}-\frac{1}{\rho}\frac{\partial\overline{p}}{\partial x_i}+\frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\overline{\tau_{ji}}-\rho\overline{u_j'u_i'}\right) \tag{10}$$

$$\rho c_p\left(\frac{\partial\overline{T}}{\partial t}+\overline{u_j}\frac{\partial\overline{T}}{\partial x_j}\right)=\frac{\partial}{\partial x_j}\left(-\overline{q_j}-\rho c_p\overline{u_j'T'}\right)+S\left(\overline{T}\right) \tag{11}$$

The above formula declares the Reynolds time-averaged equation system for incompressible fluids, laying the foundation for RANS simulation.

2.2. Direct Numerical Simulation Method for Turbulence

Direct numerical simulation of turbulence is the most stable and accurate method for simulating turbulence problems, which involves directly solving the Navier Stokes equations with sufficient spatial and temporal resolution accuracy to provide a complete model description of the required turbulence problem, including all detailed flow structures and evolution processes. However, due to the large-scale range involved in the calculation, direct numerical simulation of turbulence also requires a significant amount of computation.

Formulas 12 and 13 are the given N-S equations:

$$\frac{\partial \tilde{u}_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\bar{u}_i \bar{u}_j\right)=-\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x_i}+g_i+\nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} \quad (i, j=1,2,3) \tag{12}$$

$$\frac{\partial \tilde{u}_j}{\partial x_i}=0 \tag{13}$$

where u_i is the directional velocity component of x_i, p、ρ、v represent pressure, density, and viscosity coefficient of motion, respectively.

2.3. Large Eddy Simulation

The core idea of LES is to only simulate large-scale eddies, as they have a significant impact on the main characteristics of flow. Compared to small eddies, mathematical models are generally used to approximate their impact on large eddies, so that turbulence can be simulated more reasonably with fewer grids and fewer calculations. If the N-S equation is directly used for simulation, although the influence of small eddies on fluid motion is relatively small, this ignores the influence of small eddies on the overall fluid properties, such as diffusion to fully consider the influence of large and small eddies. LES divides the field flow into two parts by simulating a filtering function.

The filtered variables are defined as:

$$\overline{\phi}(\mathbf{x})=\int_D \phi\left(\mathbf{x}'\right) G\left(\mathbf{x}, \mathbf{x}'\right) d \mathbf{x}' \tag{14}$$

where D represents the flow field region, and G represents the function that determines the size of the filter.

$$\overline{\phi}(\mathbf{x})=\frac{1}{V} \int_V \phi\left(\mathbf{x}'\right) d \mathbf{x}', \quad \mathbf{x}' \in \mathcal{V} \tag{15}$$

where V represents the volume of the computing unit, and the filtering function G (x, x ') is defined as:

$$G\left(\mathbf{x}, \mathbf{x}'\right)\begin{cases}1 / V, & \mathbf{x}' \in \mathcal{V} \\ 0, & \mathbf{x}' \text{otherwise}\end{cases} \tag{16}$$

In Formula 16, one part is the filtered field flow, which is the large eddy part that needs to be directly simulated; Another small part of the grid is the small vortex part. Small vortices can be considered isotropic, so large and small vortices can be treated separately and calculated using a unified method.

Filter the incompressible N-S equation to obtain:

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_i}\left(\rho \bar{u}_i\right)=0 \tag{17}$$

$$\frac{\partial}{\partial t}\left(\rho \bar{u}_i\right)+\frac{\partial}{\partial x_j}\left(\rho \bar{u}_i \bar{u}_j\right)=\frac{\partial}{\partial x_j}\left(\mu \frac{\partial \pi_i}{\partial x_j}\right)-\frac{\partial \bar{p}}{\partial x_i}-\frac{\partial \tau_{i j}}{\partial x_j} \tag{18}$$

The difference between the above equation and the original flow equation is the addition of a sub-grid stress term, where sub-grid stress is defined as:

$$\tau_{i j} \equiv \rho \overline{u_i u_j}-\rho \overline{u_i} \overline{u_j} \tag{19}$$

3. Research Progress in Large Eddy Simulation

3.1. Development of Large Eddy Simulation

The turbulence large eddy simulation model was first developed by Smogorinsky [4] in the 1960s, originally for studying atmospheric problems. In 1970, Deardroff [5] first applied the LES theory to simulate channel hydraulic problems caused by pressure gradients. Since 1972, Ferziger and Reynolds have been conducting systematic research on large eddy simulations [6]. In the 1980s, most researchers shifted their attention from LES to DNS. However, during this period, Moin et al. [7] continued to study LES, proposing sub-grid models for large eddy simulation and identification methods for turbulent coherent structures. They improved the Smagorinsky model to simulate high Reynolds numbers and complex turbulence situations more accurately. Because large eddy simulation is based on the principles of coherent structures and turbulence statistical theory, improvements have been made to the time-averaged and universality issues in previous turbulence models, making them more suitable for predicting unsteady turbulence. Therefore, since the 1990s, the focus of research has returned to LES.

3.2. Theoretical Progress in Large Eddy Simulation

The United States is at the forefront of international research in large eddy simulation. Since 1973, the Turbulence Research Center at Stanford University has conducted systematic research on LES, and for the first time, the large eddy simulation method has been used for the numerical simulation of turbulent channel flow. In 1975, a research team led by Leslie proposed a dynamic sub-grid model, which can automatically adjust the model coefficients of sub-grid stress based on information from large-scale flow fields. This model greatly improves the accuracy of large eddy simulation and has become one of the widely used sub-grid models in large eddy simulation. In recent years, turbulence research teams from the Institute of Mechanics of the Chinese Academy of Sciences and Tsinghua University have made achievements and breakthroughs in large eddy simulation. Mathematical and computational scientists from Tsinghua University have developed a series of numerical algorithms and computational techniques in large eddy simulation, such as finite element method, mesh method, etc.

4. Applications of Large Eddy Simulation

Large eddy simulation is a numerical calculation method for turbulence, which was first used in atmospheric research and later used to simulate water flow. It has been widely applied in many fields such as architecture and aerospace.

4.1. Applications of Large Eddy Simulation in Architecture

Zhou et al. used large eddy simulation to simulate the low-rise building standard model TTU (Texas Institute of Technology) as the research object. In Figures 1 to 3, A-B is the windward wall, B-C is the roof, C-D is the windward wall, WT-TJ curve is the wind tunnel experiment of the Tongji University experimental team, LES-TJ curve is the LES simulation result of the Tongji University experimental team, WT-UWO curve is the wind tunnel experiment result of Xi'an University of Ontario, LES-Selvam curve is the full-scale LES simulation result of Selvam, and the measured curve is the on-site experimental result of Levitan. The experimental calculation results show that the average and fluctuating wind pressure on the surface of the building model obtained from large eddy simulations is in good agreement with the wind tunnel experiment and on-site measurement results

(Figures 1 to 3) [8]. There is no Reynolds number effect on the full-scale low-rise buildings. The dynamic wind pressure on the surface can be predicted through large eddy simulations of scaled models [8]. Zhuang et al. used large eddy simulation to simulate indoor airflow distribution. Figure 4 shows the approximate process of the development of the attached jet from the inlet to the outlet of the air supply outlet, simulated using LES under a known isothermal attached jet in a calculated room size and the position of the inlet and outlet. The experimental results [9] indicate that the large eddy simulation method not only provides more detailed information about the flow field but also demonstrates another characteristic of the large eddy simulation method, which is that the instantaneous values of the calculated variables can fully reflect the characteristics of instantaneous changes in turbulent flow [9].

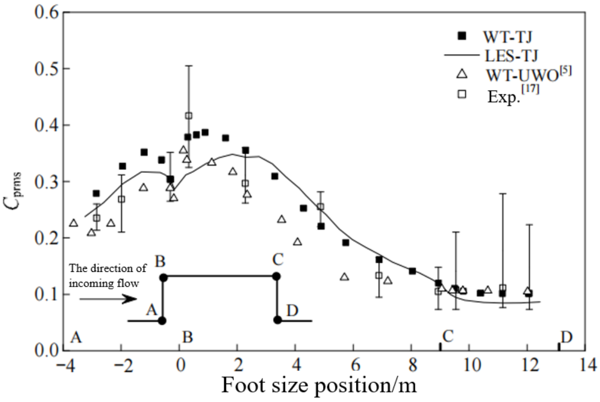

Figure 1: Average pressure coefficient [8].

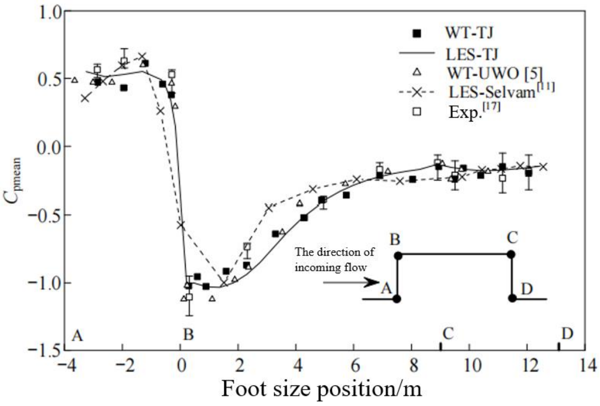

Figure 2: Fluctuating pressure coefficient [8].

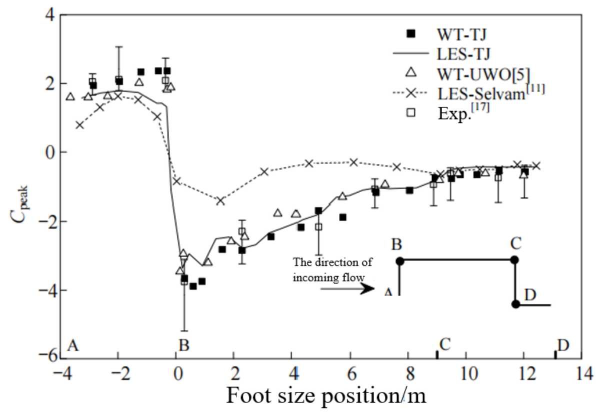

Figure 3: Peak pressure coefficient [8].

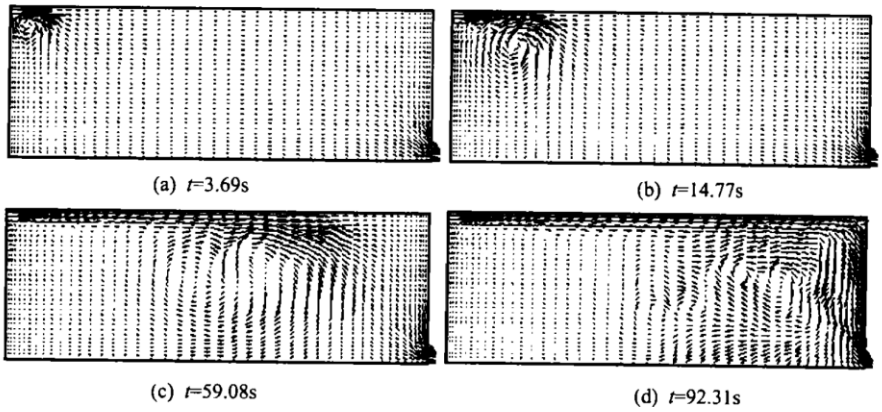

Figure 4: Instantaneous velocity vector diagram on the symmetry plane during the initial flow stage [9].

4.2. Applications of Large Eddy Simulation in Aerospace

To improve the safety of aviation flights, aviation turbulence has always been one of the key and difficult points of the aviation meteorological research center. Song conducted large eddy simulations on the impact of five types of aviation turbulence on aircraft turbulence, including wake vortices and low-level turbulence during the take-off and landing stages, as well as convective turbulence, mountain wave turbulence, and clear air turbulence during the cruising stage. Figure 5 shows the LES of a strong wind process that affected Beijing Capital and Daxing Airport in September 2022. Region a in Figure 5 shows the turbulent structure caused by high-altitude wind shear, while Region b shows the upward and downward motion forced by terrain. Region c shows the upward propagation of terrain waves. Region d represents the large-scale turbulent motion in the upper part of the boundary layer. As shown in Figure 5, the simulation shows that high-resolution LES can better clarify the source and lifecycle of aviation turbulence, significantly improving the understanding of the mechanism of aviation turbulence and the ability to diagnose and warn [10]. At present, applied research is mainly conducted in various NASA research centers and research departments of large enterprises such as Boeing. The superiority of large eddy simulation technology is reflected in its strong ability to predict turbulent structures, accurate simulation of vortex scaling and rotation, etc. Therefore, it is increasingly recognized by experimental simulations.

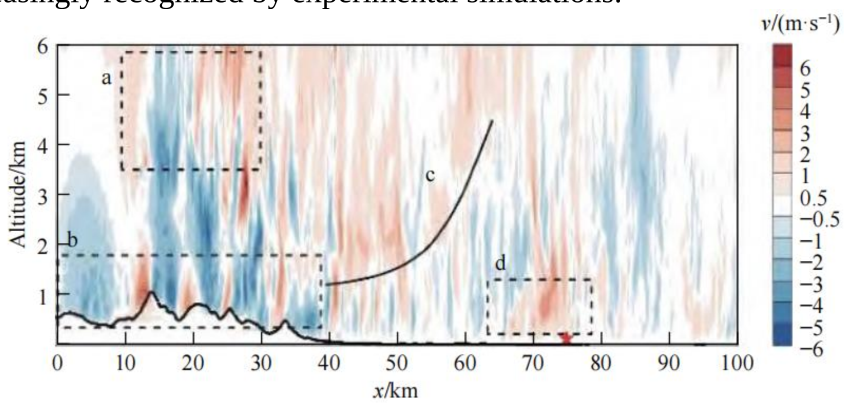

Figure 5: Vertical wind speed distribution on the west-east section of a strong wind process simulation in September 2022 [10].

5. Conclusion

In this article, the basic ideas and formulas of the Reynolds averaged method, direct numerical simulation method and large eddy simulation method are introduced. The development of large eddy simulation and its widespread applications in atmospheric science, architecture, aerospace, and other fields are analyzed.

This article focuses on summarizing the application of large eddy simulation in both architecture and aerospace. In experiments, large eddy simulation accurately and efficiently predicts the development of turbulence and simulate complex atmospheric turbulence.

Large eddy simulation, as a reliable tool for predicting turbulent motion, is rapidly developing with computational science. Large eddy simulation has incomparable advantages in predicting unsteady turbulence and high-speed turbulence, and it is also a link between the Reynolds time-averaged turbulence model and direct simulation. Although there are many challenges in large eddy simulation, its development still faces many challenges. For example, (1) How to quantify and control the numerical errors in large eddy simulation calculations, so that they do not ignore the image of the pressure filter scale model. (2) Due to the absence of inertial sub-regions in the near wall region, large eddy simulation calculations lose their advantages and degenerate into direct numerical simulations. Therefore, how to deal with near-wall turbulence is one of the important obstacles in practical applications.

Overall, with the development of theory and computer computing power, large eddy simulations can more accurately simulate problems such as turbulence. In the future, it can integrate with other scientific fields and continuously promote the development of engineering and scientific fields.

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