Journal of Electronics and Information Science (2024)
Clausius Scientific Press, Canada

DOI: 10.23977/jeis.2024.090120
ISSN 2371-9524 Vol. 9 Num. 1

Optimal line selection based on multi-beam bathymetry technology

Wubing Liang, Yun Dong*, Liqun Huang, Haixing Huang, Peicheng Zheng

Xiamen Huaxia University, Xiamen, 361024, China
**Corresponding author*

Keywords: Multi-beam Bathymetry, Sinusoidal Theorem, Coverage Width

Abstract: With the development and utilization of marine resources in China, the importance of seabed topography and landform exploration has gradually become prominent, and the advanced multi-beam bathymetry system is of great significance to improve social and economic benefits. In order to completely cover the sea area with the shortest measurement length, this paper analyzes the relationship between the coverage width of the multi-beam and the overlap rate between the two adjacent strips and the seawater depth, the opening angle of the multi-beam transducer, and the seabed slope by trigonometric functions. According to the specific sea conditions, the corresponding mathematical model is established.

1. Introduction

Multi-beam sounding system [1] is a bathymetry technology that has better performance than single-wavelength sounding systems. In a multi-beam sounding system, the signal emitted by the sensor has multiple beams, each beam with different direction and angle. After receiving the sound waves returned from the seabed, the receiving transducer can measure a certain width of the covered water depth strip with the survey line of the survey ship as the axis. In this way, the bathymetry technology can be extended from the original point and line to the surface [2], and the sea depth data can be obtained more accurately.

However, the seabed topography fluctuates greatly. If the average water depth in the sea area is used to design measurement line intervals, there may be missed measurements at shallow water depths, which affect the quality of measurement. If the measurement line spacing is designed based on the shallowest water depth in the sea area, there may be excessive overlap at deeper depths, resulting in large data redundancy and affecting measurement efficiency.

Therefor we studied the relationship between seawater depth, multi-beam bathymetry coverage width and overlap rate, established a related mathematical model, and designed a measurement scheme with the shortest measurement length, which can completely cover the entire rectangular sea area to be measured, and the overlap rate between adjacent bands is 10%~20%.

2. Model of multi-beam bathymetry

2.1 Bathymetric models for different line directions

In order to simplify the spatial relationship between the sea area and the multi-beam system, it is assumed that the transducer of the multi-beam system is located at sea level, and the coverage width of the multi-beam is the coverage width on the slope of the seabed, and the survey ship performs the survey in a steady state[3]. For ease of understanding, we converted the sea area into a cuboid.

2.1.1 Mathematical model of sea depth

Since the direction of the survey line is always perpendicular to the line where the coverage width is located in the horizontal projection, we build a three-dimensional map of ABCD-A′B′C′D′ with a diamond-shaped bottom surface to represent part of the sea area, as shown in Figure 1 below.

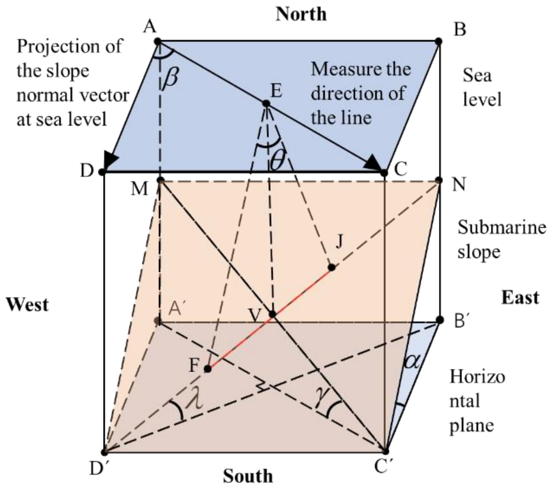

Figure 1: Schematic diagram of part of the sea area

In the Figure 1, ABCD is the sea level, MNC′D′ is the submarine slope, A′B′C′D′ is the horizontal plane, ∠CAD is the angle β between the survey line and the normal vector of the slope surface, and ∠NC′B′ is the slope α .

From Figure 1, it can be seen that the projection of the measuring ship on the seabed slope always falls on the line segment CM. Therefore, we took out the section CAA′C′ separately, as shown in Figure 2 below.

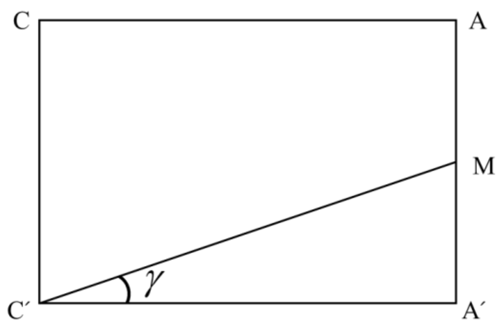

Figure 2: Schematic diagram of section CAA′C′

In this paper, the length of the bottom edge is set as the unit length of 1, as can be seen from the trigonometric relationship[4].

$$NB' = MA' = \tan\alpha$$
$$A'C' = AC = 2\cos\beta \tag{1}$$

So

$$\gamma = \arctan\left|\frac{\tan\alpha}{2\cos\beta}\right|,\left(\beta \neq \frac{\pi}{2} + n\pi, n \in Z\right) \tag{2}$$

Using the trigonometric relationship, it can be seen that.

$$a = \frac{D_0}{\tan\gamma},\left(\beta \neq \frac{\pi}{2} + n\pi, n \in Z\right) \tag{3}$$

Wherein, the distance from the center point of the survey line to the coastline is a, and the depth of the sea water at the center point of the sea area is D_0.

When $\beta = \frac{\pi}{2} + n\pi, (n \in Z)$, as can be seen from Figure 3, the seabed depth of the survey ship sailing along the survey line is the seawater depth at the center point of the sea area, which is D_0 only related to the seabed slope α.

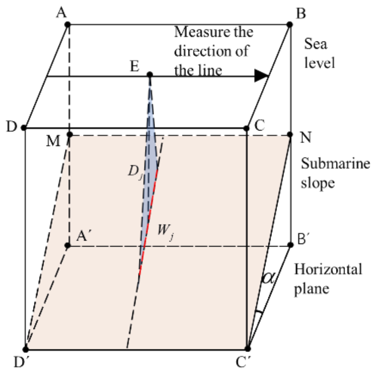

Figure 3: Schematic diagram of multi beam survey line coverage width

Similarly, using the trigonometric relationship, it can be seen that.

$$a = \frac{D_0}{\tan\alpha},\left(\beta = \frac{\pi}{2} + n\pi, n \in Z\right) \tag{4}$$

When $90° \leq \beta \leq 270°$, it is easy to obtain through similar triangles.

$$\frac{a - s}{a} = \frac{D_j}{D_0} \tag{5}$$

Wherein, s represents the distance from the surveying ship to the center point of the sea area, D_j represents the depth of the sea water when the distance between the measuring ship and the center of the sea area is s, if it is north-south, the subscript j represents the j survey line from east to west, and if it is east-west, the subscript j represents the j survey line from south to north.

It can be obtained that the sea depth when measuring the distance between the ship and the center of the sea area is s .

$$D_j = \frac{D_0 (a - s)}{a}, (90° < \beta < 270°) \tag{6}$$

When $0° \le \beta \le 90°$ or $270° \le \beta \le 360°$.

$$\frac{a+s}{a} = \frac{D_j}{D_0} \tag{7}$$

So

$$D_j = \frac{D_0 (a + s)}{a}, (0° \le \beta \le 90° \text{ or } 270° \le \beta \le 360°) \tag{8}$$

In summary, the mathematical model for measuring the seabed depth D_j when the distance between the ship and the center of the sea is s.

$$\begin{cases} \gamma = \arctan\left|\dfrac{\tan\alpha}{2\cos\beta}\right|, \left(\beta \ne \dfrac{\pi}{2} + n\pi, n \in Z\right) \\ a = \begin{cases} \dfrac{D_0}{\tan\gamma}, \left(\beta \ne \dfrac{\pi}{2} + n\pi, n \in Z\right) \\ \dfrac{D_0}{\tan\alpha}, \left(\beta = \dfrac{\pi}{2} + n\pi, n \in Z\right) \end{cases} \\ D_j = \begin{cases} \dfrac{D_0 (a - s)}{a}, (90° \le \beta \le 270°) \\ \dfrac{D_0 (a + s)}{a}, (0° \le \beta \le 90° \text{ or } 270° \le \beta \le 360°) \end{cases} \end{cases} \tag{9}$$

2.1.2 Mathematical model of multi-beam coverage width

As can be seen from Figure 1, the coverage width of the multiple beams emitted by the survey ship always falls on the line segment DM, so we took out the cross-section CAA′C′ separately, as shown in Figure 4 below:

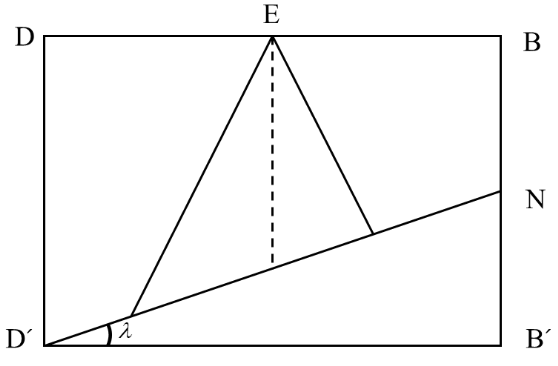

Figure 4: Schematic diagram of section DBD′B′

From the trigonometric relationship, it can be seen that.

$$NB' = \tan\alpha$$
$$D'B' = DB = 2\sin\beta \tag{10}$$

So

$$\lambda = \arctan\left|\frac{\tan\alpha}{2\sin\beta}\right|, (\beta \neq n\pi, n \in Z) \tag{11}$$

The left triangle relationship in the vertical direction of the measuring line is shown in Figure 5 below.

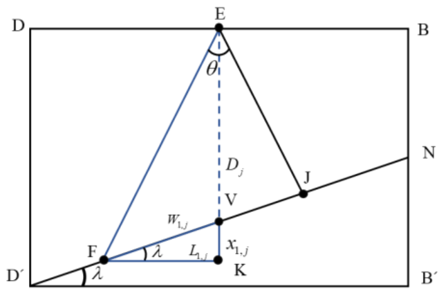

Figure 5: Schematic diagram of coverage width on the left side of the cross-section DBD′B′

From the trigonometric function, it can be seen that the projection of the left covering width $W_{1,j}$ in the horizontal plane $L_{1,j}$ is.

$$L_{1,j} = \left(D_j + x_{1,j}\right)\tan\frac{\theta}{2} \tag{12}$$

Wherein, $x_{1,j}$ represents the distance from the seabed to the horizontal plane when the distance between the measuring vessel and the center of the sea is s_j.

From the trigonometric function, $x_{1,j}$ is.

$$x_{1,j} = L_{1,j}\tan\lambda \tag{13}$$

By combining trigonometric functions, formulas （11） and （12）, the relationship between the left coverage width $W_{1,j}$ and the opening angles θ and λ of the multi beam transducer can be obtained as follows.

$$W_{1,j} = \frac{x_{1,j}}{\sin\lambda} = \frac{D_j\tan\frac{\theta}{2}\tan\lambda}{\left(1-\tan\frac{\theta}{2}\tan\lambda\right)\sin\lambda}, (\beta \neq n\pi, n \in Z) \tag{14}$$

The relationship between the triangles on the right side of the vertical direction of the survey line is shown in Figure 6 below.

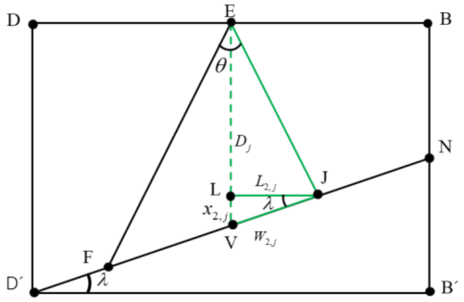

Figure 6: Schematic diagram of coverage width on the right side of the cross-section DBD′B′

From the trigonometric function, it can be seen that the projection of the left covering width $W_{2,j}$ in the horizontal plane $L_{2,j}$ is.

$$L_{2,j} = \left(D_j - x_{2,j}\right)\tan\frac{\theta}{2} \tag{15}$$

From the trigonometric function, $x_{2,j}$ is.

$$x_{2,j} = L_{2,j}\tan\lambda \tag{16}$$

By combining trigonometric functions, the relationship between the right coverage width $W_{2,j}$ and the opening angles θ and λ of the multi beam transducer can be obtained as follows.

$$W_{2,j} = \frac{x_{2,j}}{\sin\lambda} = \frac{D_j\tan\frac{\theta}{2}\tan\lambda}{\left(1+\tan\frac{\theta}{2}\tan\lambda\right)\sin\lambda}, \left(\beta \neq n\pi, n \in Z\right) \tag{17}$$

According to equations (14) and (17), it can be seen that the coverage width W_j of the multi beam when the distance between the measuring ship and the center of the sea is s_j.

$$W_j = W_{1,j} + W_{2,j}, \left(\beta \neq n\pi, n \in Z\right) \tag{18}$$

When $\beta = \frac{\pi}{2} + n\pi, \left(n \in \mathbb{Z}\right)$ as shown in Figure 7, it can be seen from the trigonometric relationship.

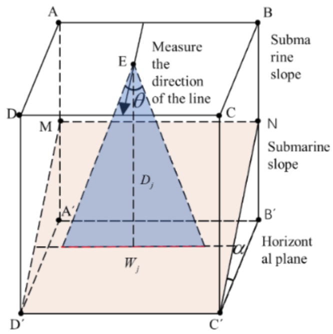

Figure 7: Schematic diagram of multi beam survey line coverage width

From the trigonometric relationship, it can be seen that.

$$W_j = 2D_j \tan\frac{\theta}{2}, (\beta = n\pi, n \in Z) \tag{19}$$

In summary, the mathematical model of the coverage width of multi-beam bathymetry W_j is.

$$\begin{cases} \lambda = \arctan\left|\dfrac{\tan\alpha}{2\sin\beta}\right|, (\beta \neq n\pi, n \in Z) \\ W_{1,j} = \dfrac{D_j \tan\frac{\theta}{2}\tan\lambda}{(1-\tan\frac{\theta}{2}\tan\lambda)\sin\lambda}, (\beta \neq n\pi, n \in Z) \\ W_{2,j} = \dfrac{D_j \tan\frac{\theta}{2}\tan\lambda}{(1+\tan\frac{\theta}{2}\tan\lambda)\sin\lambda}, (\beta \neq n\pi, n \in Z) \\ W_j = \begin{cases} W_{1,j} + W_{2,j}, (\beta \neq n\pi, n \in Z) \\ 2D_j \tan\frac{\theta}{2}, (\beta = n\pi, n \in Z) \end{cases} \end{cases} \tag{20}$$

2.2 Mathematical model of the optimal survey line

2.2.1 Mathematical model of the overlap rate between adjacent bands trending north-south

The relationship between the overlap rate and the coverage width $j, j+1$ of the survey vessel on the η first survey line W is shown in Figure 8.

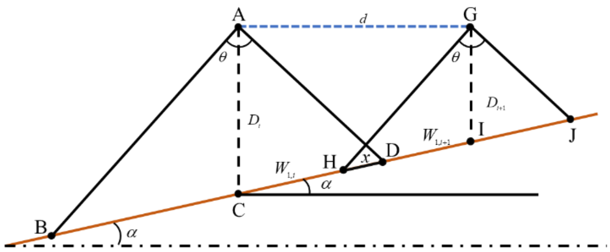

Figure 8: Schematic diagram of the overlapping width of the measuring vessel on the j and j+1 measuring lines

The overlap between adjacent bands is.

$$\eta = \frac{x}{W} = \frac{W_{2,j} + W_{1,j+1} - \dfrac{d}{\cos\alpha}}{W_j} \tag{21}$$

The mathematical model of the spacing between the two adjacent survey lines in the north-south direction is as follows.

$$d_j = \frac{(1-\eta)\times W_j \times \cos\alpha \times \left(1-\tan\left(\frac{\theta}{2}\right)\times\tan\alpha\right)\times\sin\alpha}{\left(1-\tan\left(\frac{\theta}{2}\right)\times\tan\alpha\right)\times\sin\alpha + \tan\left(\frac{\theta}{2}\right)\times\tan^2\alpha\times\cos\alpha} \tag{22}$$

Wherein d_j represents the distance between the j line and j+1 line.

2.2.2 Mathematical model of the optimal survey line

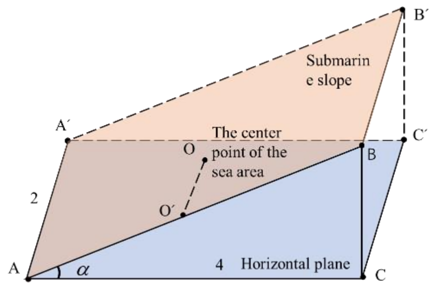

Figure 9: Schematic diagram of three-dimensional model

As shown in Figure 9, in order to obtain the measurement scheme with the shortest measurement length, which can completely cover the entire sea area to be measured, and the overlap rate between the two adjacent strips meets 10%~20%. We set the overlap rate of two adjacent lines to be 10%, let the distance between the first measuring line and segment NB' be S, where $S < W_{1,1}$ can be obtained.

$$\begin{cases} S + \sum_{j=1}^{n} d_j + W_{2,n+1} \geq AB \\ S + \sum_{j=1}^{n} d_j \leq AB \end{cases} \tag{23}$$

Using the exhaustive method [5], each S case of a gradual increase of one meter is substituted into equation (23), and all the n that meet the conditions can be obtained, the minimum value of n is added to 1 as the number of survey lines, and the first survey line is placed at the maximum value of S corresponding to the minimum value n.

Using the exhaustive method [5], substitute each case where S gradually increases by one meter into formula (23) to obtain all n that meet the conditions. Take the minimum value of n and add 1 as the number of measuring lines, and place the first measuring line at the maximum value of S corresponding to the minimum value of n.

It can be obtained that the total length of the survey line $L_{-total2}$ is.

$$L_{-total2} = (n+1)\times AA' \tag{24}$$

3. Conclusion

In summary, the model can fully cover the sea area with the shortest measurement length and meet the overlap rate requirements, effectively reducing the measurement distance and avoiding missed measurements, thereby ensuring the efficiency and accuracy of the measurement. However, in practical applications, it is necessary to consider more complex environmental conditions and make corresponding adjustments to the solution to ensure its feasibility and effectiveness in practice.

References

[1] Zhang Wei. Application of multi-beam bathymetry system in underwater topographic survey [D]. China University of Geosciences, 2009.

[2] Liu Jingnan, Zhao Jianhu. Current situation and development trend of multi-beam bathymetry system [J]. Marine Surveying and Mapping, 2002(05):3-6.

[3] Cheng Fang, Hu Yicheng. Research on optimization method of multi-beam measurement survey line layout [J]. Journal of Marine Technology, 2016, 35(02):87-91.

[4] Liang Xinghua. Knowledge reconstruction of trigonometric functions [D]. Central China Normal University, 2018.

[5] Tang Xiaojian. Exploring the Application of Exhaustive Approach in the Practice of C Language Projects [J]. Computer Programming Skills and Maintenance, 2022(04):22-25.